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in the Presence of Tax Evasion**

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# Optimal Taxes on Wealth and Consumption in the Presence of Tax Evasion

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## Abstract

This article incorporates tax evasion into an optimum taxation framework with individuals differing in earning abilities and initial wealth. We find that despite the possibility of its evasion a tax on initial wealth should supplement the optimal nonlinear income tax, given a positive correlation between initial wealth and earning abilities. Further, even if income and initial wealth are taxed optimally, it is still desirable to levy a tax on commodities, though it can be evaded as well. Thus, our result provides a rationale for a comprehensive tax system. Optimal tax rates on commodities differ in general, however for the special case of a uniform evasion technology it turns out that equal rates are optimal if preferences are homothetic and weakly separable.

*Keywords:* *Optimal Taxation, Tax Evasion*

*JEL Classification:* *D82, H21, H24, H26*

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# 1 Introduction

When designing the tax system, governments have to observe the possibility that individuals and firms try to find legal or even illegal ways in order to escape (part of) the tax liability. This reaction of the taxpayers severely restricts the extent to which a particular tax can be used as an instrument for financing publicly provided goods or for redistribution. For instance, it is often claimed that financial wealth can easily be concealed from tax authorities by moving assets offshore or by simply not reporting the true amount of wealth.<sup>1</sup> Then most notably the rich, who are intended to bear the major burden, given that wealth is distributed unequally in most societies, do not contribute as a much as one might expect. Another important issue in tax policy is the avoidance of indirect taxes, such as the value added tax in the European Union.<sup>2</sup>

In this article we analyze the question of whether taxes on wealth and on the consumption of goods are adequate instruments in an optimal tax system, if the possibility of evading them is accounted for. So far the issues of optimal taxation and tax evasion have largely been treated separately in the literature. The optimal income taxation literature starting with Mirrlees (1971) ignores with very few exceptions the problem of tax evasion, which is at odds with high levels of tax evasion observed in reality. On the other hand the tax-evasion literature starting with Allingham and Sandmo (1972) and Kolm (1973) takes the tax system as given and focuses on the taxpayer's evasion decision as well as on the determination of optimal audit rates and penalties.

We construct a framework, which allows us to analyze the trade-off between redistribution and efficiency losses due to tax evasion. For this, we extend the standard Mirrlees optimum taxation model in two ways. First, we introduce initial wealth as a second distinguishing characteristic of individuals in addition to heterogeneous ability levels. Second, we assume that taxes on initial wealth and on commodities are subject to tax evasion. The central question of the analysis is whether in such a framework an optimal nonlinear income tax should be supplemented by taxes on initial wealth and on commodities, despite the existence of tax evasion.

There are two related studies which introduce tax evasion into an optimum taxation framework and analyze the role of commodity taxes. Cremer and Gahvari (1993) also assume, as we do, that taxes on commodities but not on income can be evaded. They analyze the influence of tax evasion on optimal commodity taxes in a representative-agent model à la Ramsey. Boadway et al. (1994) analyze the optimal tax problem in a Mirrlees model, if individuals evade income taxes. They try to explain the direct-indirect tax mix observed in most countries. As indirect taxes may be

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<sup>1</sup>The existence of tax havens represents a particular opportunity for this, as is revealed by the recent debate on information transmission from foreign banks to the tax authorities in their customers' home countries.

<sup>2</sup>For a recent overview on evasion of the VAT see Keen and Smith (2006).

more difficult to evade than direct taxes and as the incentive to evade may increase with tax rates, they argue that indirect taxes can be a useful supplement in an optimal tax system. This result is similar to what we find, but in our study it is the evasion of the tax on initial wealth, which makes taxes on commodities desirable, even if the latter can be evaded as well.<sup>3</sup>

A simplifying assumption in our article is to take initial wealth as exogenously given. The most appealing interpretation of initial wealth is that it consists of inheritances received from the previous generation. However, as our model is static, it does not allow to study the dynamics behind this issue. There exist some other papers which do pay attention to the fact that initial wealth creates a second distinguishing characteristic, in addition to earning abilities. Cremer et al. (2001) also consider a static economy with exogenously given initial wealth, which is however unobservable to the social planner. They investigate the role of commodity taxes in such an economy in the presence of an optimal nonlinear income tax. Similarly, Boadway et al. (2000) and Cremer et al. (2003) study the desirability of a tax on capital income as a surrogate for the taxation of inheritances, which again are assumed to be unobservable. In contrast to these studies Brunner and Pech (2008) assume that wealth is observable to the social planner. As their model is dynamic they are able to analyze the optimal taxation of inheritances. In the present study we drop the extreme assumptions of complete observability and unobservability respectively. In our model the social planner can only observe wealth reported to tax authorities, but not the true size of initial wealth. Individuals can conceal part of their wealth from tax authorities under some cost, for example by moving assets offshore or by simply not reporting the true amount of wealth.

In the economy we describe, individuals use labor income together with initial wealth for the consumption of two goods. There exists a nonlinear income tax, a proportional tax on initial wealth and per-unit taxes on commodities. For simplicity we assume that there are only two types of individuals who differ in ability and initial wealth. The high-able individual also owns a higher amount of initial wealth, thus we assume that there is a fixed relationship between abilities and wealth. Earning abilities are unobservable to the social planner and initial wealth is only partly observable due to the existence of tax evasion. The social planner wants to redistribute from high- to low-able individuals. Regarding the income tax the planner is restricted because of the well known efficiency-equity trade-off that arises in Mirrlees' optimal tax problem with asymmetric information. In our model redistribution is also possible with taxes on initial wealth and on commodities, but the existence of tax evasion adds a restriction on redistributing with these tax

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<sup>3</sup>Cremer and Gahvari (1995) and Pestieau et al. (2004) also incorporate tax evasion into an optimum taxation framework. Both articles concentrate on the income tax and study the joint determination of optimal tax rates and audit policies.

instruments. To keep the model simple and tractable, we introduce tax evasion in a rather stylized way. Individuals and firms can evade taxes under some cost, but once they have incurred those costs, they cannot be detected by tax authorities. Thus, there is no decision under uncertainty as in the Allingham - Sandmo tax evasion model.

In a first step we analyze whether an optimal nonlinear income tax should be supplemented by a tax on initial wealth. We find that despite the existence of tax evasion a positive tax on initial wealth is always optimal in our model and that the size of the tax depends on two effects. On the one hand taxing initial wealth allows for further redistribution, as the high-able individual also owns a larger amount of wealth. On the other hand a higher tax rate leads to an increase in taxes evaded and might thus even reduce tax revenues.

In a second step we analyze the role of commodity taxes in an optimal tax system. The famous Atkinson - Stiglitz result (Atkinson and Stiglitz, 1976) states that an optimal nonlinear income tax does not need to be supplemented by commodity taxes if preferences on labor supply and consumption are weakly separable. However, this result is derived in a model where individuals differ only in earning abilities. If, additionally, individuals differ in initial wealth, Cremer et al. (2001) have shown that a role for commodity taxes arises. In contrast to our study they assume that initial wealth is completely unobservable and can therefore not be taxed at all. We find that even if initial wealth is taxed optimally and even if preferences are weakly separable, welfare can be further increased by the use of commodity taxes even though they are exposed to tax evasion.

After having shown that it is optimal to use all available tax instruments we are also interested in the optimal structure of commodity taxes. It turns out that without further simplifying assumptions not much can be said, besides that optimal commodity taxes will differ in general. To get a better understanding, we consider two special cases. First, we assume that evasion costs are uniform among commodities. There we find that uniform commodity taxes are optimal if preferences are homothetic in consumption. The second case assumes that taxes on one of the two goods cannot be evaded at all. Contrary to intuition it turns out that one cannot exclude the case that calls for taxing the good higher for which tax evasion is possible.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 analyzes the optimal taxation of initial wealth and section 4 deals with the taxation of commodities, including the analysis of the optimal structure of commodity taxes. Finally, section 4 concludes.

## 2 The model

In the economy there exist two types of individuals  $i = L, H$ . The types differ in two characteristics, namely in earning abilities  $\omega_L < \omega_H$  and exogenously given initial wealth  $e_L < e_H$ , i.e. we assume a fixed relationship between abilities and initial wealth. There is a large number of individuals of each type  $i = L, H$ ; we normalize the size of each group to one.<sup>4</sup> Individuals live for one period, in which they work and consume. By providing labor supply  $l_i$  they obtain a pre-tax income  $z_i = \omega_i l_i$ . Gross income  $z_i$  is subject to a nonlinear income tax and the resulting net income is denoted by  $x_i$ . Initial wealth is taxed at a proportional rate  $\tau_e$ . Individuals use after-tax income  $x_i$  together with net initial wealth for the consumption  $c_{ji}$  of two commodities  $j = 1, 2$ , which are subject to per-unit taxes denoted by  $\tau_j$ . Commodities are produced by a large number of firms in two industries, with perfect competition among the identical firms in each industry. Again the size (the set of firms) of each industry is normalized to one. Labor is the only input to production and technologies are assumed to be linear. Quantities are chosen in such a way that the (constant) marginal costs of production are equal to one for both commodities.

### *Individual behavior*

Taxes on initial wealth and on commodities are exposed to tax evasion. First, consider the households' concealment technology for the wealth tax. An individual of type  $i$  conceals a fraction  $\alpha_{ei}$  of initial wealth from tax authorities at the expense of concealment costs depending on the fraction  $\alpha_{ei}$ . Once individuals have incurred those costs they cannot be detected by tax authorities, thus there is no decision under uncertainty in the model. Concealment costs depend on the evaded amount in the following way:<sup>5</sup> concealing one unit of  $e_i$  entails a resource cost described by the function  $K_{ei}(\alpha_{ei})$ , which is assumed to be an increasing and convex function of the evaded share  $\alpha_{ei}$ , with  $K_{ei}(0) = 0$ . Total concealment costs for individual  $i$  are then equal to  $\alpha_{ei} e_i K_{ei}(\alpha_{ei})$ . To simplify notation we define  $k_{ei}(\alpha_{ei}) \equiv \alpha_{ei} K_{ei}(\alpha_{ei})$ .<sup>6</sup>

Both types of individuals have the same strictly concave utility function,  $u(c_{1i}, c_{2i}, l_i)$ , with  $\partial u / \partial c_{ji} > 0$ ,  $\partial u / \partial l_i < 0$ . Individuals maximize their utility subject to the private budget constraint

$$p_1 c_{1i} + p_2 c_{2i} \leq x_i + R_{ei} e_i \tag{1}$$

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<sup>4</sup>More formally, the members of each group are represented by the real numbers in the interval  $[0, 1]$ , whose size (Lebesgue measure) is one.

<sup>5</sup>Cremer and Gahvari (1993) and Boadway et al. (1994) model tax evasion in a similar manner.

<sup>6</sup>Note that convexity of  $K_{ei}(\alpha_{ei})$  implies strict convexity of  $k_{ei}(\alpha_{ei})$  because  $\partial^2 k_{ei}(\alpha_{ei}) / \partial \alpha_{ei}^2 = 2 \partial K_{ei}(\alpha_{ei}) / \partial \alpha_{ei} + \alpha_{ei} \partial^2 K_{ei}(\alpha_{ei}) / \partial \alpha_{ei}^2 > 0$ .

with consumer prices denoted by  $p_j$ ,  $j = 1, 2$ , and where  $R_{ei} \equiv 1 - \tau_e(1 - \alpha_{ei}) - k_{ei}(\alpha_{ei})$ . For an individual  $i$ ,  $R_{ei}e_i$  represents the amount that remains after the deduction of the tax on the reported wealth and of the evasion costs; thus  $R_{ei}e_i$  can be interpreted as her effective net initial wealth, which together with net labor income  $x_i$  can be spent on consumption.

From the first-order-condition for the optimal  $\alpha_{ei}$  one gets

$$\tau_e = \partial k_{ei}(\alpha_{ei}) / \partial \alpha_{ei}, \quad (2)$$

i.e. individuals conceal wealth until the marginal cost of concealment equals its marginal benefit. Observe that the optimal fraction  $\alpha_{ei}$  is determined solely by the tax rate and is independent of total initial wealth  $e_i$ , which follows from the assumption that per-unit evasion costs depend only on the fraction  $\alpha_{ei}$ . Due to convexity  $\partial \alpha_{ei}(\tau_e) / \partial \tau_e = (\partial^2 k_{ei}(\alpha_{ei}) / \partial \alpha_{ei}^2)^{-1} > 0$  holds. That is, the model has the realistic property that the fraction  $\alpha_{ei}$  of hidden wealth rises with the tax rate.<sup>7</sup> Moreover, note that we allow the evasion technology to differ among individuals, hence in general they evade different fractions of  $e_i$ .

#### *Firm behavior*

Commodity taxes are levied on firms, which have access to an evasion technology similar to that of the households. Each firm of industry  $j$  evades commodity taxes by concealing a fraction  $\alpha_j$  of its sales again at a cost depending on the evaded fraction  $\alpha_j$ . Let  $c_j$  denote the produced quantity of a firm in industry  $j$ . Concealing one unit of good  $j$  entails a resource cost described by the function  $K_j(\alpha_j)$ , with  $\partial K_j(\alpha_j) / \partial \alpha_j > 0$ ,  $\partial^2 K_j(\alpha_j) / \partial \alpha_j^2 \geq 0$  and  $K_j(0) = 0$ . Again we define  $k_j(\alpha_j) \equiv \alpha_j K_j(\alpha_j)$ . Then the maximization problem of a firm in industry  $j$  can be written as

$$\max \pi_j = p_j c_j - c_j - (1 - \alpha_j) \tau_j c_j - c_j k_j(\alpha_j). \quad (3)$$

The right-hand side of (3) consists of sale revenues minus the cost of production (remember that the constant marginal costs are equal to one), minus taxes paid to tax authorities and minus the

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<sup>7</sup>This is in accordance with a large majority of empirical studies on this subject. See among others Clotfelter (1983), Andreoni et. al. (1998) and Pudney et. al. (2000). It contrasts the expected-utility model of income tax evasion due to Allingham and Sandmo (1972), where evaders are penalized if detected by tax authorities. In this model the effect of an increase in marginal tax rates on tax evasion is ambiguous, if individuals are risk averse and if absolute risk aversion decreases with income. Yitzhaki (1974) even shows that if the penalty is proportional to the amount of taxes evaded - and not to the amount of income evaded as in the Allingham-Sandmo model - tax evasion will even decrease when the tax rate rises.

cost of tax evasion. Solving the first order condition for the optimal fraction  $\alpha_j$  one gets

$$\tau_j = \partial k_j(\alpha_j)/\partial \alpha_j, \quad (4)$$

thus firms also evade taxes until the marginal cost of concealment equals its marginal benefit. As before the optimal  $\alpha_j$  is determined solely by the tax rate  $\tau_j$  and is thus independent of the amount  $c_j$  sold by the firm. Also  $\partial \alpha_j(\tau_j)/\partial \tau_j > 0$  still holds. In equilibrium firms supply good  $j$  at a consumer price

$$p_j = 1 + \tau_j(1 - \alpha_j) + k_j(\alpha_j), \quad (5)$$

with  $\alpha_j$  chosen optimally according to (4). The corresponding producer price is equal to the constant marginal costs (equal to one). Observe that without tax evasion the consumer price is equal to  $1 + \tau_j$ . Hence, all gains of tax evasion ( $\tau_j \alpha_j - k_j(\alpha_j)$ ) are transmitted to consumers via a lower consumer price, and firms make zero profits (as usual under the assumption of a linear technology). Finally, the equilibrium quantity of each commodity  $j = 1, 2$  is determined by the aggregate demand function  $c_{jL} + c_{jH}$ .

#### *The social planner's problem*

In a next step we describe the social planner's maximization problem. For this purpose we first introduce the indirect utility function of household  $i$  for given tax rates  $\tau_e, \tau_1, \tau_2$  and given net and gross income  $x_i, z_i$ ,

$$v_i(x_i, z_i, e_i, \tau_e, \tau_1, \tau_2) \equiv \max \{u(c_{1i}, c_{2i}, z_i/\omega_i) \mid p_1 c_{1i} + p_2 c_{2i} \leq x_i + R_{ei} e_i\}. \quad (6)$$

First-best taxes referring to abilities are not implementable, because abilities are not observable to the social planner. Nor can the social planner infer abilities from initial wealth, as only reported wealth is observable.<sup>8</sup> Therefore the tax authority imposes an income tax as a second-best instrument. As there are no restrictions on the functional form of the income tax the standard way of solving such a problem is to maximize a social welfare function with respect to the individuals' net

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<sup>8</sup>One may argue that with a fixed relationship between abilities and initial wealth the social planner could in principle identify individuals by observing reported wealth, and impose a lump-sum tax on abilities. This would make the high-able individual worse off than the low-able individual (Mirrlees, 1974) and, thus, be a further incentive for the H type to conceal wealth. We avoid this complexity by assuming that the social planner can commit not to impose a lump-sum tax on abilities by using information transmitted through reported wealth. Note, that a related problem arises also in the standard Mirrlees optimum income tax model (Mirrlees, 1971): Given a tax schedule, gross income of high-able individuals is higher than gross income of low-able individuals. Thus, ex-post the social planner could identify individuals as well.



and gross income bundles  $(x_L, z_L)$ ,  $(x_H, z_H)$ , subject to a self-selection constraint and a resource constraint. By this the optimal income tax for the two types of individuals is determined implicitly as the difference  $z_i - x_i$ ,  $i = L, H$ . The other available tax instruments  $\tau_e$ ,  $\tau_1$  and  $\tau_2$  are taken as fixed at some rate for the moment.

The utilitarian social welfare function, which is the objective function of the maximization problem, reads

$$\max_{x_i, z_i, i=L, H} f_L v_L(x_L, z_L, e_L, \tau_e, \tau_1, \tau_2) + f_H v_H(x_H, z_H, e_H, \tau_e, \tau_1, \tau_2), \quad (7)$$

where  $f_L$  and  $f_H$ , with  $f_L \geq f_H \geq 0$ , represent the weights of the two types of individuals. As usual we assume that preferences fulfill the condition of "agent monotonicity" (Seade, 1982). Formally this means that  $MRS_{zx}^L > MRS_{zx}^H$  holds at any vector  $(x, z)$ , where  $MRS_{zx}^i$  is defined as  $MRS_{zx}^i \equiv -(\partial v_i / \partial z_i) / (\partial v_i / \partial x_i)$ . This assumption - also known as the single crossing condition - implies that for any income tax function the high-able individual does not choose to earn less income than the low-able.<sup>9</sup>

Whereas the objective function is standard, the resource constraint has to be modified in our setting. It reads

$$x_L + x_H \leq z_L + z_H + \tau_e((1 - \alpha_{eL})e_L + (1 - \alpha_{eH})e_H) + \tau_1(1 - \alpha_1)(c_{1L} + c_{1H}) + \tau_2(1 - \alpha_2)(c_{2L} + c_{2H}) - g. \quad (8)$$

The social planner has to collect tax revenues in order to finance public spending  $g$ . One can see that the base for a tax on initial wealth is reported wealth and not the true amount of wealth. The same holds for the base of commodity taxes  $\tau_j, j = 1, 2$ . As reported wealth and reported consumption decrease with increasing tax rates, an increase of  $\tau_e$ ,  $\tau_1$  and  $\tau_2$  might even reduce tax revenues. For the taxation of initial wealth the possibility of a Laffer effect only arises due to the existence of tax evasion. This is not the case for the taxation of a single good  $j$ , where a Laffer effect might also arise without the possibility of tax evasion due to the existence of substitution effects. Further note that resources spent on the concealment activity represent pure waste, as they are not included in the resource constraint.

The self-selection constraint is again standard. We restrict the analysis to cases, where the social planner wants to redistribute from high- to low-ability persons and, due to the assumption

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<sup>9</sup>It should be mentioned that the existence of initial endowments makes the assumption of agent-monotonicity more problematic than in the standard case. The reason is that with  $e_H$  sufficiently larger than  $e_L$  the high-able individual might demand at least as much additional net income as compensation for achieving an additional unit of gross income as the low-ability individual.

of agent monotonicity, only the self-selection constraint, which prevents the high-able individual from mimicking the low-able individual (i.e. from choosing the bundle which is intended for the low-able) is binding in the optimum. The self-selection constraint reads

$$v_H(x_H, z_H, e_H, \tau_e, \tau_1, \tau_2) \geq v_H(x_L, z_L, e_H, \tau_e, \tau_1, \tau_2). \quad (9)$$

Maximizing (7) subject to (8) and (9) with respect to  $x_i$  and  $z_i$  yields the first-order conditions for the optimal bundles of net and gross income. The Lagrange multipliers of the resource constraint and the self-selection constraint are denoted by  $\lambda$  for the former and by  $\mu$  for the latter. The marginal utility of income of the high-able individual in the case of mimicking is described by  $\partial v_H[L]/\partial x_L$ . The first-order conditions for  $x_i$  and  $z_i$  read

$$f_L \frac{\partial v_L}{\partial x_L} - \lambda + \lambda \tau_1(1 - \alpha_1) \frac{\partial c_{1L}}{\partial x_L} + \lambda \tau_2(1 - \alpha_2) \frac{\partial c_{2L}}{\partial x_L} - \mu \frac{\partial v_H[L]}{\partial x_L} = 0, \quad (10)$$

$$f_H \frac{\partial v_H}{\partial x_H} - \lambda + \lambda \tau_1(1 - \alpha_1) \frac{\partial c_{1H}}{\partial x_H} + \lambda \tau_2(1 - \alpha_2) \frac{\partial c_{2H}}{\partial x_H} + \mu \frac{\partial v_H}{\partial x_H} = 0, \quad (11)$$

$$f_L \frac{\partial v_L}{\partial z_L} + \lambda + \lambda \tau_1(1 - \alpha_1) \frac{\partial c_{1L}}{\partial z_L} + \lambda \tau_2(1 - \alpha_2) \frac{\partial c_{2L}}{\partial z_L} - \mu \frac{\partial v_H[L]}{\partial z_L} = 0, \quad (12)$$

$$f_H \frac{\partial v_H}{\partial z_H} + \lambda + \lambda \tau_1(1 - \alpha_1) \frac{\partial c_{1H}}{\partial z_H} + \lambda \tau_2(1 - \alpha_2) \frac{\partial c_{2H}}{\partial z_H} + \mu \frac{\partial v_H}{\partial z_H} = 0. \quad (13)$$

The optimal income tax is described implicitly by these conditions for given tax rates  $\tau_1, \tau_2, \tau_e$  (possibly zero). This is the starting point for the analysis in the next two sections. Given an optimal income tax we want to find out whether taxes on initial wealth and on commodities can further increase social welfare, despite the presence of tax evasion. Clearly, in an economy without tax evasion a tax on exogenously given wealth would be lump-sum and therefore a desirable tax instrument for redistributive reasons, given that ability and wealth are positively correlated. The same holds true for a uniform expenditure tax, which Brunner and Pech (2008) have shown to be equivalent to a tax on initial wealth in an economy without tax evasion. How those results have to be modified due to the existence of tax evasion will be shown in the following sections.

### 3 Optimal taxation of initial wealth

In this section we concentrate on the taxation of initial wealth, therefore commodity taxes are assumed to be zero. When analyzing whether a proportional tax on initial wealth is a useful supplement to an optimal income tax the social planner has to take two aspects into account. On the one hand a tax on initial wealth allows for further redistribution, as the high-able individual also owns a larger amount of wealth. But on the other hand a higher tax rate leads to an increase in taxes evaded and might thus even reduce tax revenues. Proposition 1 addresses this trade-off faced by the social planner. Let the optimal value function of the maximization problem (7)-(9) be denoted by  $S(\tau_e, \tau_1, \tau_2)$ .

**Proposition 1:** The welfare effect of a marginal increase of a tax  $\tau_e$  on initial wealth, given that  $x_i, z_i$  are adapted optimally, is described by

$$\frac{\partial S}{\partial \tau_e} = \mu \frac{\partial v_H[L]}{\partial x_L} ((1 - \alpha_{eH})e_H - (1 - \alpha_{eL})e_L) - \lambda \tau_e (e_L \frac{\partial \alpha_{eL}}{\partial \tau_e} + e_H \frac{\partial \alpha_{eH}}{\partial \tau_e}). \quad (14)$$

Despite the existence of tax evasion a positive tax rate on initial wealth is always optimal in our model, as  $\frac{\partial S}{\partial \tau_e} |_{\tau_e=0} > 0$ .

**Proof:** The derivation of equation (14) is provided in the Appendix.

One can see that the welfare effect of a marginal increase of  $\tau_e$  consists of two different parts with opposite signs. The first term on the right-hand side of equation (14) describes the effect on the self-selection constraint. It is unambiguously positive given that the high-able individual reports a higher amount of wealth to tax authorities than the low-able individual. That is, if  $(1 - \alpha_{eH})e_H > (1 - \alpha_{eL})e_L$  an increase of  $\tau_e$  relaxes the self-selection constraint.<sup>10</sup> To understand the intuition behind this mechanism, it helps to split it up into two steps. Assume in a first step that after a marginal increase of  $\tau_e$  each individual  $i$  can be fully compensated for the loss of net initial wealth  $\partial(R_{ei}e_i)/\partial \tau_e = -(1 - \alpha_{ei})e_i$ <sup>11</sup> by an increase of net income of the same amount. This makes mimicking less attractive, as long as the compensation  $(1 - \alpha_{ei})e_i$  is higher for the high-able individual, and relaxes the self-selection constraint. Then in a second step further redistribution via the income tax becomes possible and this in turn increases social welfare.

<sup>10</sup>Observe that if  $\alpha_{eL} \geq \alpha_{eH}$  the effect becomes zero only if both individuals evade all their wealth. If, however,  $\alpha_{eH} > \alpha_{eL}$  it becomes zero for positive amounts of wealth reported to tax authorities.

<sup>11</sup>Use the definition for  $R_{ei}$  following (1) and apply the envelope theorem.

However, due to tax evasion individuals cannot be fully compensated, as an increase of  $\tau_e$  also increases the fraction of wealth concealed by individuals. This is taken into account by the second part of the RHS of (14), which affects the resource constraint. It is always negative except for a zero tax rate, where it obviously is zero. It can be interpreted as the deadweight loss of tax evasion. The intuition behind this effect is that an increase of  $\tau_e$  leads to a decline in the fraction of wealth reported and therefore has a negative influence on attainable tax revenues. Clearly this occurs due to our formulation of tax evasion, where the proportion of hidden wealth increases with the tax rate.

Although individuals have the possibility to conceal wealth from tax authorities, a positive tax on initial wealth is always optimal in our model, as  $\frac{\partial S}{\partial \tau_e} |_{\tau_e=0} > 0$ . This is due to the fact that the effect on the self-selection constraint is positive at  $\tau_e = 0$ , because  $\alpha_{ei} = 0$  and  $e_H > e_L$ , while the effect on the resource constraint is zero at  $\tau_e = 0$ .

Next we turn to the characterization of the optimal tax on initial wealth. Obviously the optimum occurs when  $\frac{\partial S}{\partial \tau_e} = 0$ , thus it is obtained by setting the RHS of (14) equal to zero, which is the first-order condition of the maximization problem (7)-(9) for  $\tau_e$ . Intuitively, the social planner should increase  $\tau_e$  as long as the positive redistributive effect is larger than the negative deadweight loss effect and set the optimal tax rate  $\tau_e^*$  such that both effects have the same size.

We know from above that the optimal tax rate is greater than zero. On the other hand, it cannot be optimal to set such a high tax rate that both types of individuals conceal all their wealth, i.e.  $\alpha_{ei} = 1$  for  $i = L, H$ , because then government revenues are the same as at a tax rate of zero while evasion costs are wasted. Obviously, it depends on the evasion cost functions, whether individuals choose to conceal all their wealth. This case cannot occur if marginal cost of evasion become prohibitively high, i.e. if  $\partial k_{ei}(1)/\partial \alpha_{ei} = \infty$ . Otherwise there exists a threshold  $\hat{\tau}_{ei}$  for each individual  $i$  such that  $\alpha_{ei}(\tau_e) = 1$  for all  $\tau_e \geq \hat{\tau}_{ei}$  and  $\alpha_{ei}(\tau_e) < 1$  for all  $\tau_e < \hat{\tau}_{ei}$ . That is,  $\hat{\tau}_{ei}$  is the lowest tax rate at which individual  $i$  conceals all her wealth. One finds that the optimal tax rate fulfills  $\tau_e^* < \hat{\tau}_{eH}$ . This is immediate if  $\hat{\tau}_{eL} \leq \hat{\tau}_{eH}$ , because then both individuals would conceal everything at a tax rate  $\tau_e \geq \hat{\tau}_{eH}$ . Further if  $\hat{\tau}_{eL} > \hat{\tau}_{eH}$ , it turns out that (14) is negative at  $\hat{\tau}_{eH}$  (this follows from  $\alpha_{eL}(\hat{\tau}_{eH}) < 1$  and  $\alpha_{eH}(\hat{\tau}_{eH}) = 1$ , while  $\partial \alpha_{ei}/\partial \tau_e$  at  $\hat{\tau}_{eH}$  is positive for  $i = L$  and zero for  $i = H$ ), which tells us that the optimum tax rate  $\tau_e^*$  must be lower than  $\hat{\tau}_{eH}$ .

In general, given the optimal tax rate we cannot determine the sign of  $R_{ei}(\tau_e^*) = 1 - \tau_e^*(1 - \alpha_{ei}) - k_{ei}(\alpha_{ei})$ . In order to exclude the implausible case that effective net wealth is negative, we assume that evasion costs fulfill the condition  $k_{ei}(1) \leq 1$ ,  $i = L, H$ . That is, the total cost  $e_i k_{ei}(1)$  for an individual  $i$  of concealing all initial wealth does not exceed  $e_i$ .

*Lemma 1:* If  $k_{ei}(1) \leq 1$ , then  $R_{ei}(\tau_e) \geq 0$  for all  $\tau_e$  and  $R_{ei}(\tau_e) > 0$  for  $\tau_e < \hat{\tau}_{ei}$ .

**Proof:** First, observe that for all  $\tau_e \geq \hat{\tau}_{ei}$ ,  $R_{ei}(\tau_e) = 1 - k_{ei}(1)$  (remember that  $\alpha_{ei}(\tau_e) = 1$  for all  $\tau_e \geq \hat{\tau}_{ei}$ ), which is nonnegative, if  $k_{ei}(1) \leq 1$ . For all  $\tau_e < \hat{\tau}_{ei}$  we have  $\partial R_{ei}(\tau_e)/\partial \tau_e = -(1 - \alpha_{ei}) < 0$ . Altogether this proves the Lemma. Q.E.D.

Knowing that given the condition  $k_{ei}(1) \leq 1$ , effective net wealth is always nonnegative, we conclude for the high-able individual that  $R_{eH} > 0$  holds at the optimum tax rate  $\tau_e^*$ , as  $\tau_e^* < \hat{\tau}_{eH}$  (from above). However, for the low-able individual we cannot exclude that  $R_{eL} = 0$  at the optimal tax rate  $\tau_e^*$ . The latter situation arises if  $\tau_e^* \geq \hat{\tau}_{eL}$  and  $k_{eL}(1) = 1$ .<sup>12</sup>

Still whether  $R_{eH} \geq R_{eL}$  at the optimal tax rate  $\tau_e^*$  depends on the evasion technology of the individuals. If the marginal cost of tax evasion  $\partial k_{ei}(\alpha_{ei})/\partial \alpha_{ei}$  is larger for the low-able individual at any  $\alpha_{ei}$ ,  $R_{eH} > R_{eL}$  holds as then the per-unit rent of tax evasion  $\tau_e \alpha_{ei} - k_{ei}(\alpha_{ei})$  is larger for the  $H$  type. Obviously, in that case the effective net wealth  $R_{ei}e_i$  is higher for the  $H$  type at the optimal tax rate. If, however, the marginal cost of tax evasion is lower for the  $L$  type, then we have  $R_{eL} > R_{eH}$  and the effective net wealth could arise to be higher for the  $L$  type. We abstract from this case and assume for the rest of the analysis that  $R_{eH}e_H > R_{eL}e_L$  holds. There are two arguments which corroborate this assumption. First, it is plausible that wealthier individuals also have an at least as good access to tax evasion activities as the low-able, implying lower (or equal) marginal cost of tax evasion and  $R_{eH} \geq R_{eL}$ . Second, even in the opposite case of  $R_{eL} > R_{eH}$  the inequality  $R_{eH}e_H > R_{eL}e_L$  may hold, given  $e_H > e_L$ . Altogether, the case of  $R_{eH}e_H > R_{eL}e_L$  is certainly the more realistic scenario.

## 4 Optimal taxation of commodities

A classical result on the role of indirect taxes in an optimal tax system is due to Atkinson and Stiglitz (1976). They showed that when preferences are weakly separable in labor supply and consumption, nonlinear income taxation does not need to be supplemented by commodity taxes. However, this result is derived in a model, where individuals differ in only one characteristic, namely in earning abilities. In a more recent paper Cremer et al. (2001) have shown that even if preferences are weakly separable between labor and consumption, commodity taxation is a useful

<sup>12</sup>Note that  $(1 - \alpha_{eL})$  enters (14) with opposite sign compared to  $(1 - \alpha_{eH})$ , therefore the argument for  $\tau_e^* < \hat{\tau}_{eH}$  does not apply for the low-able individual.

instrument of tax policy if individuals differ not only in abilities but also in endowments. The role of commodity taxation in their model is to tax indirectly initial endowments which are assumed to be unobservable. This is in contrast to our model where endowments are partly observable and where reported endowments are subject to taxation. We prove in the following that even if endowments are taxed optimally and in addition individuals can evade commodity taxes, there still remains a role for commodity taxes in an optimal tax system. Furthermore, we analyze the role of tax evasion on the optimal structure of commodity taxes, similar to Cremer and Gahvari (1993). Their study analyses the influence of tax evasion on optimal taxes in a representative agent model à la Ramsey and is thus, in contrast to our study, not concerned with redistribution.

The welfare effect of commodity taxation is determined by differentiating the optimal value function of (7)-(9) with respect to  $\tau_j$ .

**Proposition 2:** The welfare effect of a marginal increase of a tax  $\tau_j$  on commodity  $j = 1, 2$ , given that  $x_i, z_i$  are adapted optimally and that  $\tau_e$  is chosen optimally is described by

$$\begin{aligned} \frac{\partial S}{\partial \tau_j} = & \mu \frac{\partial v_H[L]}{\partial x_L} (1 - \alpha_j)(c_{jH}[L] - c_{jL}) - \lambda \tau_j \frac{\partial \alpha_j}{\partial \tau_j} (c_{jL} + c_{jH}) \\ & + \lambda (1 - \alpha_j) \sum_{k=1}^2 \tilde{\tau}_k \left( \frac{\partial c_{kL}^{com}}{\partial p_j} + \frac{\partial c_{kH}^{com}}{p_j} \right). \end{aligned} \quad (15)$$

Given that commodity  $j$  is a normal good and that preferences are weakly separable between labor supply and consumption, welfare can be further increased by an introduction of a commodity tax, as  $\frac{\partial S}{\partial \tau_j} |_{\tau_j=0} > 0$ .

**Proof:** The derivation of equation (15) is provided in the Appendix.

In formula (15), Hicksian compensated demand for good  $k$  is denoted by  $c_{ki}^{com}$  and  $\tilde{\tau}_k \equiv \tau_k(1 - \alpha_k)$ . Proposition 2 states that despite the existence of tax evasion positive commodity taxes on normal goods are optimal in our model, even if preferences between labor supply and consumption are weakly separable. From equation (15) one can see that the total welfare effect of a marginal increase of  $\tau_j$  consists of three effects, one redistributive effect on the self-selection constraint (multiplier  $\mu$ ) and two efficiency effects on the resource constraint (multiplier  $\lambda$ ). First, consider the effect on the self-selection constraint. Observe that in the case of normal goods the high-able individual when mimicking consumes more than the low-able individual, i.e.  $c_{jH}[L] > c_{jL}$ , because of  $R_{eHeH} > R_{eLeL}$ . Consequently a marginal increase of  $\tau_j$  hurts the mimicker more than the

mimicked individual, as long as  $\alpha_j < 1$  (firms report a positive amount of sales, thus the consumer price increases with  $\tau_j$ ).<sup>13</sup> In other words mimicking becomes less attractive, which relaxes the self-selection constraint. Hence, the redistributive effect of  $\tau_j$  is positive.

The efficiency effects in equation (15) describe the deadweight loss due to tax evasion and the distorting effects on compensated demand. The interpretation of the deadweight loss effect induced by tax evasion, which is described by the second term on the right hand side of (15), is quite similar to the one given in the preceding section for the case of wealth taxes. Higher tax rates lead to an increase of the fraction of hidden sales and therefore reduce the tax base for the commodity tax. Finally, the last term in equation (15) represents the effects on compensated demand associated with the distortion of the consumer price  $p_j$  due to an increase of  $\tau_j$ . Both efficiency effects are of second order, thus for a zero tax rate they are zero, while the effect on the self-selection constraint is positive at  $\tau_j = 0$ . Hence an introduction of a commodity tax increases welfare.<sup>14</sup> To summarize, a role for commodity taxes arises in our model even if they are exposed to tax evasion and even if exogenous initial wealth is taxed optimally. Thus in the economy we describe, it is optimal to supplement the income tax by all other available tax instruments. The reason is that this allows to balance the deadweight loss effects created by tax evasion.

It is interesting to note that in an economy without initial wealth  $c_{jH}[L] = c_{jL}$  holds, if preferences are weakly separable in labor supply and consumption. Then one obviously returns to the classical Atkinson-Stiglitz result.<sup>15</sup>

After having shown that commodity taxation increases welfare we now turn to the discussion of the optimal structure of commodity taxes. We consider the case of normal goods. The optimal tax structure is described by the following relationship, which is obtained by setting the first-order conditions for  $\tau_1$  and  $\tau_2$  from equation (15) equal to zero:

$$\frac{c_{1H}[L] - c_{1L}}{c_{2H}[L] - c_{2L}} = \frac{\frac{\tau_1}{1-\alpha_1} \frac{\partial \alpha_1}{\partial \tau_1} (c_{1L} + c_{1H}) - \sum_i (\tilde{\tau}_1 \frac{\partial c_{1i}^{com}}{\partial p_1} + \tilde{\tau}_2 \frac{\partial c_{2i}^{com}}{\partial p_1})}{\frac{\tau_2}{1-\alpha_2} \frac{\partial \alpha_2}{\partial \tau_2} (c_{2L} + c_{2H}) - \sum_i (\tilde{\tau}_1 \frac{\partial c_{1i}^{com}}{\partial p_2} + \tilde{\tau}_2 \frac{\partial c_{2i}^{com}}{\partial p_2})}. \quad (16)$$

It turns out that insights into the optimal tax structure are rather limited. It depends on the relative size of the redistributive and efficiency effects, which in turn depend on preferences and on the evasion technologies for both commodity taxes. However, as the size of those effects remains

<sup>13</sup>Note that  $\partial p_j / \partial \tau_j = 1 - \alpha_j$ . With  $\alpha_j = 1$  we have  $p_j = 1 + k_j(1)$  and a further increase of  $\tau_j$  has no effect.

<sup>14</sup>For an inferior good the introduction of a subsidy improves welfare, as then  $c_{jH}[L] < c_{jL}$ .

<sup>15</sup>Observe that in the Atkinson-Stiglitz model a case for a positive (negative) tax arises if the good is a complement (substitute) to leisure. In our model the argument for taxing good  $j$  would be reinforced if it were a complement to leisure and weakened if it were a substitute.

rather arbitrary it is impossible to draw any precise conclusion on the optimal structure of  $\tau_1$  and  $\tau_2$  for the general case. What one can see is that uniform tax rates will only arise by coincidence, in general optimal tax rates will differ. To gain more insight we now turn to two special cases. The first one deals with uniform evasion costs for both commodities, which implies that for  $\tau_1 = \tau_2$  firms would conceal the same fraction  $\alpha_j$  of  $c_1$  and  $c_2$ . The second special case assumes that only the tax on one of the two goods can be evaded.

#### 4.1 *Uniform evasion costs for both commodity taxes*

Assume now that evasion costs are uniform, i.e.  $k_1(\alpha_1) = k_2(\alpha_2)$ , the cost of concealing a fraction  $\alpha_j$  is the same for both goods. This special case allows us to draw some conclusions under what circumstances uniform commodity taxation is optimal.

**Proposition 3:** Given uniform evasion costs for both commodities, uniform commodity taxes are optimal if preferences for  $c_1$  and  $c_2$  are homothetic and weakly separable between consumption and labor supply.

**Proof:** The derivation of proposition 3 is provided in the Appendix.

Clearly with identical evasion costs a motive for differential commodity taxation can only arise from some asymmetry of the households' preferences with respect to the two commodities. Proposition (3) states that for homothetic preferences (i. e., when the relative importance of the two goods does not vary with the available budget) a uniform commodity tax is optimal. This result is especially interesting as it is related to a standard result of the optimum taxation literature, which states that if income is subject to an optimal linear income tax, uniform commodity taxation is optimal if preferences between consumption and labor supply are weakly separable and if Engel curves for goods are linear (see for example Deaton, 1979). We find that if individuals differ in initial wealth and if commodity taxes can be evaded at a uniform cost, preferences have to be weakly separable between consumption and labor and homothetic in consumption for uniform commodity taxes to be optimal, even if income can be taxed nonlinearly. Note that in our model initial wealth exists and can only be taxed at a proportional rate, which is the analogy to the restriction in Deaton (1979) that income can only be taxed linearly.

One can show that with Stone-Geary preferences, which allow to model a luxury and a necessity good in a simple way, our model would call for taxing the luxury good higher than the necessity



good. This is in contrast to the standard result mentioned above, because with Stone-Geary preferences Engel curves are still linear.<sup>16</sup>

#### 4.2 Taxes on good two cannot be evaded

Assume now that taxes on good 2 cannot be evaded because marginal evasion costs are infinitely high, i.e.  $k'_2(0) = \infty$ . A possible illustration for such a scenario could be that  $c_1$  represents services while  $c_2$  represents goods, as it is plausible that it is easier to evade taxes on services than on goods. Clearly, the assumption that a tax on some good cannot be evaded at all is too strict, but it helps to illustrate the point we want to make. Intuitively one might expect that such a situation would call for taxing the commodity that cannot be evaded higher than the commodity for which tax evasion is possible. However, it turns out that this need not be the case as one also has to take into account the distorting effects on compensated demand. This can be seen from equation (17), which is an adapted version of equation (16). Note that now we have  $\alpha_2 = 0$  for all  $\tau_2$ :

$$\frac{c_{1H}[L] - c_{1L}}{c_{2H}[L] - c_{2L}} = \frac{\frac{\tau_1}{1-\alpha_1} \frac{\partial \alpha_1}{\partial \tau_1} (c_{1L} + c_{1H}) - \sum_i (\tilde{\tau}_1 \frac{\partial c_{1i}^{com}}{\partial p_1} + \tau_2 \frac{\partial c_{2i}^{com}}{\partial p_1})}{-\sum_i (\tilde{\tau}_1 \frac{\partial c_{1i}^{com}}{\partial p_2} + \tau_2 \frac{\partial c_{2i}^{com}}{\partial p_2})} \quad (17)$$

Optimal tax rates for  $\tau_1$  and  $\tau_2$  have to satisfy this condition. We can conclude that given normal goods the right-hand side of (17) must be positive. It is well-known that compensated demand has the property (homogeneity) that  $\sum_j p_j \frac{\partial c_{ji}^{com}}{\partial p_k} = 0$ , for  $k = 1, 2$  and any  $i = L, H$ . This and negativity (positivity) of own (cross, resp.) compensated price effects imply that the summation terms  $\sum_i (\tilde{\tau}_1 \frac{\partial c_{1i}^{com}}{\partial p_k} + \tau_2 \frac{\partial c_{2i}^{com}}{\partial p_k})$ ,  $k = 1, 2$ , in the numerator and denominator, respectively, of the RHS of (17) have opposite signs for arbitrary  $\tilde{\tau}_1, \tau_2$ . They clearly cannot be zero. This in turn means that the denominator must be positive because otherwise the RHS of (17) would be negative (the numerator would be positive). Next,  $\sum_i (\tilde{\tau}_1 \frac{\partial c_{1i}^{com}}{\partial p_2} + \tau_2 \frac{\partial c_{2i}^{com}}{\partial p_2}) < 0$  implies, again due to homogeneity, that  $\frac{\tilde{\tau}_1}{\tau_2} \leq \frac{p_1}{p_2}$ . Finally, using (5) and  $\alpha_2 = 0$ , we get  $\tau_1(1 - \alpha_1) \leq \tau_2(1 + k_1(\alpha_1))$ , which obviously holds for  $\tau_1 = \tau_2$  as  $\alpha_1 < 1$  and  $k_1(\alpha_1) > 0$ . Thus, without specifying in more detail the cost function  $k_1(\alpha_1)$  and preferences we cannot tell whether  $\tau_1 \gtrless \tau_2$  is optimal.

<sup>16</sup>For Stone-Geary preferences the sub-utility function of  $c_1, c_2$  reads  $u(c_1, c_2) = (c_1 - \gamma_1)^{a_1} (c_2 - \gamma_2)^{a_2}$ , where  $\gamma_j > 0$  and  $0 < a_j < 1$ . The parameter  $\gamma_j$  can be interpreted as the subsistence level of consumption for good  $j$  and  $a_j/(a_1 + a_2)$  as the marginal budget share.

## 5 Conclusion

In this paper we have extended the standard model of optimum income taxation by an important aspect, which conforms to reality: individuals differ not only in earning abilities, but also in initial wealth. The government can impose a rather comprehensive set of taxes: a nonlinear tax on labor income and proportional taxes on wealth and on commodities. Moreover, we have introduced the restriction that the latter two taxes can - at some cost - be evaded by individuals and firms, resp. We analyzed the question of whether there is a role for these taxes in a welfare-maximizing tax system.

It turned out that, given the essential condition that abilities and initial wealth are positively correlated, a tax on wealth - in addition to an optimal nonlinear income tax - is desirable, even if it can be evaded. Further, even if income and wealth are taxed optimally, taxes on commodities still raise social welfare, given that consumption increases with income. Thus, the result in the Atkinson-Stiglitz model, that an optimal income tax does not need to be supplemented by commodity taxes if preferences are weakly separable between labor and consumption, does not arise in our model. The main reason for this clearly comes from the existence of initial wealth as a second characteristic, which distinguishes individuals and calls for redistribution via the wealth tax. As the deadweight loss of evasion is of second order, it can be disregarded as long as the tax rate is not too high. On the other hand, a tax on commodities can, in principle, perform the same task as the tax on wealth (Brunner and Pech, 2008). However, due to the second-order effect of tax evasion it is optimal to impose both proportional taxes in our model, instead of only one, because then the overall deadweight loss is smaller.

Thus, our model provides, in a realistic framework, a rationale for the existence of a comprehensive tax system, as we find it in most industrialized economies. Clearly, it would be interesting to extend our model and consider the possibility that also the income tax can be evaded. We abstained from this in order to keep the model tractable. However, the result by Boadway et al. (1994) that in the standard Mirrlees model a case for indirect taxes arises given the income tax can be evaded while the indirect taxes cannot, indicates that our results should remain valid even with evasion of the income tax.

In our model, the optimal structure of commodity taxes depends on the individuals' preferences and on the evasion technology, but also on the differences in initial wealth. Specifically, the reference case that uniform commodity taxes are optimal arises, if evasion costs are identical for all goods and if preferences over the consumption goods are homothetic (independent of wealth difference). As a further special case we considered the situation when only the tax on one of the two goods

can be evaded. Contrary to intuition it turns out that it may be optimal to tax the good for which evasion is possible higher than the other good.

## Appendix

### *Proof of Proposition 1*

The Lagrangian for the maximization problem (7)-(9) reads

$$\begin{aligned}
L = & f_L v_L(x_L, z_L, e_L, \tau_e, \tau_1, \tau_2) + f_H v_H(x_H, z_H, e_L, \tau_e, \tau_1, \tau_2) - \lambda(x_L + x_H - z_L - z_H - \\
& \tau_e((1 - \alpha_{eL})e_L + (1 - \alpha_{eH})e_H) - \tau_1(1 - \alpha_1)(c_{1L} + c_{1H}) - \tau_2(1 - \alpha_2)(c_{2L} + c_{2H}) + g) \\
& + \mu(v_H(x_H, z_H, e_L, \tau_e, \tau_1, \tau_2) - v_H(x_L, z_L, e_L, \tau_e, \tau_1, \tau_2)). \tag{A1}
\end{aligned}$$

To abbreviate notation we write  $v_H[L] \equiv v_H(x_L, z_L, e_L, \tau_e, \tau_1, \tau_2)$ . Using the envelope theorem we get for the optimal value function  $S(\tau_e, \tau_1, \tau_2)$

$$\begin{aligned}
\frac{\partial S}{\partial \tau_e} = & f_L \frac{\partial v_L}{\partial \tau_e} + f_H \frac{\partial v_H}{\partial \tau_e} + \lambda((1 - \alpha_{eL})e_L + (1 - \alpha_{eH})e_H) - \lambda \tau_e \left( \frac{\partial \alpha_{eL}}{\partial \tau_e} e_L + \frac{\partial \alpha_{eH}}{\partial \tau_e} e_H \right) + \\
& \lambda \tau_1(1 - \alpha_1) \left( \frac{\partial c_{1L}}{\partial \tau_e} + \frac{\partial c_{1H}}{\partial \tau_e} \right) + \lambda \tau_2(1 - \alpha_2) \left( \frac{\partial c_{2L}}{\partial \tau_e} + \frac{\partial c_{2H}}{\partial \tau_e} \right) + \\
& \mu \frac{\partial v_H}{\partial \tau_e} - \mu \frac{\partial v_H[L]}{\partial \tau_e}. \tag{A2}
\end{aligned}$$

In a next step we use  $\frac{\partial v_i}{\partial \tau_e} = \frac{\partial R_{ei}}{\partial \tau_e} e_i \frac{\partial v_i}{\partial x_i}$ ,  $\frac{\partial v_H[L]}{\partial \tau_e} = \frac{\partial R_{eH}}{\partial \tau_e} e_H \frac{\partial v_H[L]}{\partial x_L}$  and  $\frac{\partial c_{ji}}{\partial \tau_e} = \frac{\partial R_{ei}}{\partial \tau_e} e_i \frac{\partial c_{ji}}{\partial x_i}$ , where  $\frac{\partial R_{ei}}{\partial \tau_e} = -(1 - \alpha_{ei})$ . Plugging those expressions into (A2) and substituting for  $f_i \frac{\partial v_i}{\partial x_i}$  from (10) and (11) yields equation (14) in the text.

### *Proof of Proposition 2*

We make again use of the envelope theorem. The derivative of the optimal value function  $S(\tau_e, \tau_1, \tau_2)$  from the maximization problem represented by the Lagrangian in (A1) with respect to  $\tau_j$  reads

$$\begin{aligned}
\frac{\partial S}{\partial \tau_j} = & f_L \frac{\partial v_L}{\partial \tau_j} + f_H \frac{\partial v_H}{\partial \tau_j} + \lambda[(1 - \alpha_j - \tau_j \frac{\partial \alpha_j}{\partial \tau_j})(c_{jL} + c_{jH}) + (1 - \alpha_j) \tau_j (\frac{c_{jL}}{\partial \tau_j} + \frac{c_{jH}}{\partial \tau_j}) \\
& + \tau_k(1 - \alpha_k) (\frac{c_{kL}}{\partial \tau_j} + \frac{c_{kH}}{\partial \tau_j})] + \mu (\frac{\partial v_H}{\partial \tau_j} - \frac{\partial v_H[L]}{\partial \tau_j}), \tag{A3}
\end{aligned}$$

with  $j \neq k$ . We find  $\frac{\partial v_i}{\partial \tau_j} = -\frac{\partial p_j}{\partial \tau_j} c_{ji} \frac{\partial v_i}{\partial x_i}$ , with  $\frac{\partial p_j}{\partial \tau_j} = (1 - \alpha_j)$ . Note also that  $\frac{\partial c_{ji}}{\partial \tau_j} = \frac{\partial c_{ji}}{\partial p_j} \frac{\partial p_j}{\partial \tau_j}$ . By

use of these expressions and substituting for  $f_i \frac{\partial v_i}{\partial x_i}$  from (10) and (11), (A3) can be transformed to

$$\begin{aligned} \frac{\partial S}{\partial \tau_j} &= \mu \frac{\partial v_H[L]}{\partial x_L} (1 - \alpha_j)(c_{jH}[L] - c_{jL}) - \lambda \tau_j \frac{\partial \alpha_j}{\partial \tau_j} (c_{jL} + c_{jH}) \\ &\quad + \lambda (1 - \alpha_j) \left[ \tilde{\tau}_j \sum_{i=L}^H \left( \frac{\partial c_{ji}}{\partial p_j} + c_{ji} \frac{\partial c_{ji}}{\partial x_i} \right) + \tilde{\tau}_k \sum_{i=L}^H \left( \frac{\partial c_{ki}}{\partial p_j} + c_{ji} \frac{\partial c_{ki}}{\partial p_j} \right) \right] \end{aligned} \quad (\text{A4})$$

By use of the Slutsky equation one gets equation (15) in the text.

### **Proof of Proposition 3**

First, observe that with  $k_1(\alpha_1) = k_2(\alpha_2)$  for  $\alpha_1 = \alpha_2$ , we have  $p_1 = p_2$  (use (5)) and  $\tilde{\tau}_1 = \tilde{\tau}_2$ , if  $\tau_1 = \tau_2$ . As is well known, compensated demand is homogeneous, thus for  $j = 1, 2$  we have

$$p_1 \frac{\partial c_{1i}^{com}}{\partial p_j} + p_2 \frac{\partial c_{2i}^{com}}{\partial p_j} = 0 \quad (\text{A5})$$

for any  $p_1, p_2 \geq 0$ . In particular  $p \left( \frac{\partial c_{1i}^{com}}{\partial p_j} + \frac{\partial c_{2i}^{com}}{\partial p_j} \right) = 0$  for  $p_1 = p_2 \equiv p$ , thus  $\tilde{\tau} \left( \frac{\partial c_{1i}^{com}}{\partial p_j} + \frac{\partial c_{2i}^{com}}{\partial p_j} \right) = 0$ , for  $\tilde{\tau}_1 = \tilde{\tau}_2 \equiv \tilde{\tau}$ . This means that on the RHS of (16) the effects on compensated demand are zero. Moreover, for  $\tau_1 = \tau_2$  we have  $\frac{\tau_1 \frac{\partial \alpha_1}{\partial \tau_1}}{1 - \alpha_1} = \frac{\tau_2 \frac{\partial \alpha_2}{\partial \tau_2}}{1 - \alpha_2}$ , thus (16) reduces to

$$\frac{c_{1H}[L] - c_{1L}}{c_{2H}[L] - c_{2L}} = \frac{c_{1L} + c_{1H}}{c_{2L} + c_{2H}}. \quad (\text{A6})$$

Finally, if preferences of each individual  $i$  for good 1 and 2 are homothetic and weakly separable between consumption and labor, each individual  $i$  spends the same constant share  $g_j$  of her budget  $b_i$  on each commodity  $j = 1, 2$ . Then (A6) can be rewritten as

$$\frac{g_1(b_H[L] - b_L)}{g_2(b_H[L] - b_L)} = \frac{g_1(b_L + b_H)}{g_2(b_L + b_H)}, \quad (\text{A7})$$

which is obviously true. Altogether we have shown that with uniform evasion costs, weakly separable and homothetic preferences the optimality condition (16) is fulfilled for  $\tau_1 = \tau_2$ .

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