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Abstract

This articles studies the optimal tax mix (taxes on income and commodities) under asymmetric information in a two-type model, when individuals make relative consumption comparisons. The model includes both positional and nonpositional goods, taking into account the fact that relative concerns matter for some but not for all commodities. We find that in general the whole tax system is affected by the externalities caused by the consumption of positional goods, notably also the taxes on income and on a nonpositional good. The tax rates on positional goods are higher than in the absence of status effects, reflecting their Pigouvian role. The sign of the Pigouvian part in the income tax schedule is ambiguous and depends crucially on whether status goods are complements or substitutes to leisure.

Keywords: Optimal Taxation, Externalities, Relative Consumption

JEL Classification: D62, H21, H23

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1 Introduction

This paper studies the optimal mix of a nonlinear income tax and proportional commodity taxes in the presence of relative consumption concerns. Most part of the optimal taxation literature assumes that the utility of individuals depends only on their own consumption of goods and leisure. However, there is increasing empirical evidence suggesting that individuals value not only their own absolute consumption but also their relative consumption with respect to others.[1] Moreover, there is evidence indicating that some goods are more positional than others (e.g. Alpizar et al., 2005 and Solnick and Hemenway, 2005). Typically, visible forms of consumption such as clothing or housing tend to be more positional than less visible forms such as food or insurance consumption. In accordance with this empirical evidence we construct a model that allows us to analyze the optimal tax structure assuming that some but not all commodities are positional.

The idea that individuals care about their relative position in society was first pronounced by Veblen (1899), but his view was soon displaced by the simpler neoclassical theory of consumer behavior. In an important work Dusenberry (1949) reintroduced relative preferences to consumer theory and established the relative income hypothesis, which states that the utility an individual derives from a given consumption level depends strongly on its relative magnitude in the society. Since the late 70's a literature on optimal policy issues when relative consumption matters, has gradually developed (e.g. Boskin and Sheshinski, 1978; Layard, 1980; Oswald, 1983; Ireland, 2001; Wendner and Goulder, 2008; Aronsson and Johansson-Stenmann, 2008, 2010; Micheletto, 2010).[2] It has become evident from these studies that several standard results concerning optimal tax policy and public good provision do not hold or at least have to be adapted if one takes status effects into account. For example, Aronsson and Johansson-Stenmann (2008) show that the optimal income tax schedule differs significantly from the one in the conventional case, implying substantially higher marginal income tax rates. However, this literature typically assumes that there is only one consumption good and thus, does not differentiate between positional and nonpositional forms of consumption, as we do.

Our paper is also related to a different strand of the literature that studies optimal mixed taxation (income and commodities) in an asymmetric information setting a la Mirrlees (1971) in the presence of consumption externalities (Pirttilä and Tuomala, 1997; Cremer et al., 1998; Kopczuk, 2003; Micheletto, 2008). When consuming status goods individuals impose a utility loss

[1] See e.g. Easterlin (2001), Johansson-Stenman et al. (2002), Ferrer-i-Carbonell (2005), Luttmer (2005), Carlsson et al. (2007) and Card et al. (2010). A survey of the literature is provided by Clark et al. (2008).

[2] Relative consumption concerns were also introduced in models analyzing growth (e.g. Corneo and Jeane, 1997, 2001; Wendner, 2010) or asset pricing (Abel, 1999; Dupor and Liu, 2003).

on others by increasing the reference level to which individuals compare their own consumption to, which can be viewed as a consumption externality. One important result from these previous studies is that the so called 'additivity property' first discovered by Sandmo (1975) also carries over to the more general optimal mixed tax case with heterogeneous agents, at least if the externality is of the 'atmospheric' type.[3] The 'additivity property' states that an externality is best addressed by imposing a tax directly on the externality-generating good while the rest of the tax system should be unaffected by the externality. However, in a recent paper Micheletto (2008) has shown that for the 'additivity property' to be valid it is essential that different types are equally effective as externality-generating units, i.e. it should not matter which individual increases the level of the externality with his/her consumption (as it is the case if the externality is of the 'atmospheric' type). This is typically fulfilled for environmental externalities but needs not necessarily be the case for positional externalities, where it is plausible that different agents have varying reference groups and where the identity of the consumer whose consumption increases the reference level might in fact be relevant.[4] We confirm this result and show that in general the whole tax system is affected by the positional externalities.

To the best of our knowledge, the present paper is the first one that studies jointly optimal income and commodity taxation in the presence of both positional and nonpositional goods, paying attention to the fact that some but not all commodities are positional. This is of relevance because the mixed tax case, in which income is taxed on a nonlinear scale, while commodity taxes are restricted to be linear, corresponds closely to most actual tax systems in developed countries. Thus, it seems to be the appropriate framework for a complete analysis of the influence of relative consumption concerns on the optimal tax structure. Further, we extend previous work on optimal mixed taxation and consumption externalities to a multi-externality setting. So far the literature has confined the analysis to cases with only one externality-generating good. An exception is Eckerstorfer (2011) who studies the optimal income and commodity tax structure in a multi-externality model, where however, in contrast to the present study, the externalities are restricted to be of the 'atmospheric' type.

From a policy perspective a main result of our model is that differential taxation of commodities is welfare improving and that status goods should be taxed at a higher rate. This is not surprising since in our framework the consumption of status goods generates a negative externality, and hence

[3]The term 'atmospheric' externality was introduced by Meade (1952). It is used when the externality depends on the total consumption of a particular good.

[4]For example, if one thinks of upward-comparison, meaning that individuals have a tendency to refer upwards in the income distribution, then the consumption of the rich gets a larger weight in the formation of the reference level.

taxing these goods internalizes the externalities. This point has already been raised amongst others by Frank (1999, 2008) who has, however, concerns about the political feasibility of a tax system that taxes status goods at a different rate. We share this point only to some extent, as various tax systems include certain forms of luxury taxation.[5] For example, Australia levies a luxury car tax which has to be paid at the time of the purchase if the price of a new car exceeds a certain threshold. Another example is the mansion tax in the State of New York that is levied on Real Estate property at the time of a transfer when the consideration for the entire conveyance exceeds \$ 1 million.[6] A popular argument in the political debate in favor of taxes on luxury goods is that they hurt the rich and are hence a redistributive instrument. However, Atkinson and Stiglitz (1976) have taught us that under a mild separability assumption between goods and leisure all redistribution should be carried out through the labor income tax. Hence, according to this result differential taxation of luxuries is not efficient for redistributive reasons. But in this paper we provide a different argument in favor of taxing status goods, one that has rarely found recognition in the political debate.

We proceed in the following way. First, we set up a generalized version of the two-group optimal tax model (Stiglitz, 1982) where individuals differ in earning abilities and tastes. Taste differences are reflected by different reference levels for the two types. As is common in the optimal taxation literature, earning abilities are private information to the individual. Hence, first-best taxation of abilities is not feasible which is why the government has to use a general income tax as a second-best instrument. There are three consumption goods in the economy. Two of them are positional goods and one good is a nonpositional good, where as usual only absolute consumption matters.[7] Reference levels to which individuals compare their own consumption are formed by a weighted average of total consumption of a positional good. Our formulation of the reference levels is flexible enough to include several variants discussed in the literature such as upward or within-group comparison. Each commodity is subject to a proportional tax rate, i.e. the tax system consists of a nonlinear income tax and proportional commodity tax rates.

In a next step we present our results on the optimal commodity tax structure. We derive an implicit solution for the optimal tax rates on both positional and nonpositional goods. An

[5] In most cases luxury goods and status goods coincide. Therefore, we consider taxation of status goods as a form of luxury taxation.

[6] Other OECD countries that have certain forms of luxury taxation include Denmark, Hungary and Turkey. A famous example for a luxury tax in the USA mentioned in Bagwell and Bernheim (1996) was created by the Omnibus Budget Reconciliation Act of 1990. A 10% excise tax was imposed on the portion of the retail price of certain items that exceeded a product specific threshold, which was \$30000 for cars, \$100000 for boats and yachts, \$250000 for aircraft, and \$10000 for jewelry. The luxury taxes on boats, aircraft and jewelry were repealed in 1993, the one on cars in 2002.

[7] In principle the model could be extended to include $n+m$ commodities. But in order to keep the model as simple as possible we decided to work with only three commodities, which already makes the analysis quite complex.

interesting implication of our multi-externality assumption is the result that the Pigouvian elements in the tax formulas depend on both externalities, implying that the interdependence between the positional commodities is of importance. Further, we show that in general both the tax rates on the positional as well as on the nonpositional goods depend in a complex way on the Pigouvian elements. Thus, it is optimal to use the whole set of commodity taxes to internalize the externalities, which violates the 'additivity property'. If, however, the consumption of the high- and the low-able individuals is weighted in the same way in the formation of the reference levels, then the tax rate on the nonpositional good is not affected by the externalities and the tax formulas on the positional goods simplify substantially. The same is true if there are no compensated cross price effects and/or individuals react in the same way to relative price changes.

Finally, we also discuss the optimal income tax schedule. We find that in general also the income tax is affected by the externalities, which also is in contradiction to the 'additivity property'. But again, if the consumption of the two income groups is weighted in the same way in the formation of the reference levels then the optimal marginal income tax rates contain no Pigouvian elements. We also discuss the optimal income tax schedule if commodity taxes are restricted to be uniform for example due to political economy reasons. In that case only the income tax can be used to correct for the externalities. In contrast to the previous literature that studies the optimal income tax in the presence of relative consumption concerns (e.g. Aronsson and Johansson-Stenman, 2008) we differentiate between positional and nonpositional forms of consumption. This implies that the sign of the Pigouvian part in the income tax schedule depends crucially on whether the positional goods are complements or substitutes to leisure. Moreover, we show that if the consumption of the rich is weighted higher in the formation of the reference levels (upward comparison) then with a mild additional assumption the Pigouvian part in the income tax schedule is progressive.

The rest of the paper is organized as follows. In section 2 we present the model and the maximization problem of the households and the government. In section 3 we discuss the optimal commodity tax structure and in section 4 the results on the optimal income tax schedule. Section 4 provides some discussion of the results.

2 The model

We consider an economy consisting of two types of individuals $i = L, H$, who differ in earning abilities $\omega_L < \omega_H$. The size of the population is normalized to one and π_i represents the proportion of individuals of type i in the population. By providing labor supply l_i individuals earn gross income

$z_i = \omega_i l_i$. Gross income z_i is subject to a nonlinear income tax and the resulting net income is denoted by x_i. Individuals spend their net income on the consumption of three commodities c_j, $j = 1, 2, 3$, which are produced with a linear technology with labor as the only input to production. Quantities are chosen in such a way that the (constant) marginal costs of production are equal to one, i.e. the producer prices of all commodities are equal to one.

The consumers' problem and relative concerns

In our model commodities $k = 1, 2$ are assumed to be positional goods, i.e. for these goods individuals care not only about their absolute value of consumption but also about their relative consumption with respect to others.[8] Commodity 3 on the other hand is a nonpositional good where only absolute consumption matters. As is common in the literature we assume that for the positional commodities each individual compares his/her own consumption to some reference consumption level which is determined by

$$\overline{c_{kL}} = \alpha_{kL}(L)\pi_L c_{kL} + \alpha_{kH}(L)\pi_H c_{kH} \tag{1}$$

for the L type and by

$$\overline{c_{kH}} = \alpha_{kL}(H)\pi_L c_{kL} + \alpha_{kH}(H)\pi_H c_{kH} \tag{2}$$

for the H type, with $k = 1, 2$. As in previous work we assume that the reference consumption levels are treated as exogenous to the individual. The intuition for this assumption is that individuals consider their own contribution to the reference levels as extremely small.

Highly relevant for our analysis later on will be the nonnegative weights $\alpha_{kL}(i)$ and $\alpha_{kH}(i)$, $k = 1, 2$ and $i = L, H$. Observe that the reference levels of the two types differ in general, i.e. an L type compares his/her own consumption to a different reference level than an H type. Only if $\alpha_{ki}(L) = \alpha_{ki}(H)$ the reference levels of the two types are identical. Further note that the consumption of the two types is allowed to be weighted differently in the formation of the reference level as in general $\alpha_{kL}(i) \neq \alpha_{kH}(i)$. This means that the consumption of the H types might be weighted higher (lower) in the formation of the reference levels than the consumption of the L types.

In its simplest form where all weights are equal to one, reference consumption levels are equal

[8] We use the index $j = 1, 2, 3$ when we talk about all commodities and the index $k = 1, 2$ when we talk about the positional commodities.

to the average consumption of the specific commodity in the economy. In that case both types face the same reference levels and their consumption is weighted equally in the formation of $\overline{c_{ki}}$, $k = 1, 2$ and $i = L, H$. However, our formulation of the reference levels is also flexible enough to implement other variants discussed in the literature, such as upward comparison or within-group comparison, just by adapting weights properly.[9]

Following earlier contributions to the literature (e.g., Akerlof, (1997); Corneo and Jeanne, (1997); Aronsson and Johansson-Stenman, (2008, 2010)) relative concerns are modeled as the difference between own consumption and the reference consumption levels $\overline{c_{1i}}$ and $\overline{c_{2i}}$, i.e. by $\Delta_{1i} = c_{1i} - \overline{c_{1i}}$ and $\Delta_{2i} = c_{2i} - \overline{c_{2i}}$, $i = L, H$. Thus, in addition to own consumption and labor supply, Δ_{1i} and Δ_{2i} become part of the utility function. Both types have the same strictly concave utility function, $u(c_{1i}, c_{2i}, c_{3i}, l_i, \Delta_{1i}, \Delta_{2i})$, with first partial derivatives being positive with respect to $c_{1i}, c_{2i}, c_{3i}, \Delta_{1i}, \Delta_{2i}$ and negative with respect to l_i.[10] This implies that the utility of an individual i decreases with $\overline{c_{1i}}$ and $\overline{c_{2i}}$. Thus, the consumption of good 1 and 2 generates what we call a positional externality, since individuals do not take into account the effect their own consumption of commodities 1 and 2 has on others via the reference levels. That is, when consuming positional commodities individuals increase the reference consumption levels, which worsens the relative position of other individuals.

The individuals' maximization problem is broken into two steps. In a first step, an individual i allocates a fixed amount of net income x_i over the consumption goods. Consumer prices are denoted by $p_j = 1 + \tau_j$, $j = 1, 2, 3$, i.e. the government imposes a proportional commodity tax τ_j on each of the three commodities. Maximizing utility subject to the private budget constraint yields conditional indirect utility for given gross and net income:

$$v_i(x_i, z_i, p_1, p_2, p_3, \overline{c_{1i}}, \overline{c_{2i}}) \equiv \max_{c_{1i}, c_{2i}, c_{3i}} \left\{ u(c_{1i}, c_{2i}, c_{3i}, z_i/\omega_i, \Delta_{1i}, \Delta_{2i}) \left| p_1 c_{1i} + p_2 c_{2i} + p_3 c_{3i} \leq x_i \right. \right\}. \tag{3}$$

Solving the private maximization problem described by (3) yields conditional demand functions

$$c_{ji} = c_{ji}(x_i, z_i, p_1, p_2, p_3, \overline{c_{1i}}, \overline{c_{2i}}), \tag{4}$$

[9] Upward comparison means that individuals compare themselves to other individuals above them in the income distribution whereas within-group comparison means that individuals compare themselves to individuals in the same income group.

[10] As in most other studies leisure is assumed to be nonpositional, which is justified by a number of empirical results (e.g. Carlsson et al. (2007)). An exception is Aronsson and Johansson-Stenman (2009), who assume that individuals care about both relative consumption and relative leisure.

$j = 1, 2, 3$ and $i = L, H$. Observe that in general the demand for all commodities depends on the reference levels $\overline{c_{1i}}$ and $\overline{c_{2i}}$.

In a second step, individuals determine their optimal labor supply by maximizing conditional indirect utility $v_i(x_i, z_i, p_1, p_2, p_3, \overline{c_{1i}}, \overline{c_{2i}})$ subject to the budget equation $x_i = z_i - T(z_i)$, where $T(z_i)$ denotes the nonlinear income tax function. Assuming that the income tax function is differentiable and letting the marginal income tax rate be denoted by $T'(z_i)$, individuals choose their labor supply such that

$$T'(z_i) = 1 + \frac{\partial v_i / \partial z_i}{\partial v_i / \partial x_i}. \tag{5}$$

The government's problem

The government's objective is to design a tax system, consisting of a general income tax and proportional commodity taxes τ_1, τ_2, τ_3, that maximizes a weighted sum of the individuals' utility subject to a resource constraint and a self-selection constraint, and that takes into account the positional externalities induced by the consumption of goods 1 and 2. The problem of finding the optimal income tax schedule can equivalently be stated by determining the optimal gross and net income bundles x_i, z_i for each type. Thus, the optimal income tax for the two types of individuals is determined implicitly as the difference $z_i - x_i$, $i = L, H$. Note that the available tax instruments are completely determined by the information structure of the model. Type specific first-best lump-sum taxes are not feasible because earning abilities ω_i and labor supply l_i are not observable to the government. Only gross income z_i and the distribution of types are observable, which is why the government has to use a nonlinear income tax as a second-best instrument. Further, consumption is assumed to be observable only in the aggregate, while individual consumption levels are private information. In other words the government does not know who buys how much of what good. As a consequence nonlinear commodity taxes are not feasible, and the government can tax consumption only at a proportional rate.

The utilitarian social welfare function, which is the objective function of the maximization problem of the government, reads

$$\max_{\tau_1, \tau_2, \tau_3, x_i, z_i, \overline{c_{1i}}, \overline{c_{2i}}, i=L,H} f_L v_L(x_L, z_L, p_1, p_2, p_3, \overline{c_{1L}}, \overline{c_{2L}}) + f_H v_H(x_H, z_H, p_1, p_2, p_3, \overline{c_{1H}}, \overline{c_{2H}}), \tag{6}$$

where f_L and f_H, with $f_L \geq f_H \geq 0$, represent the weights of the two types of individuals including the fractions π_L and π_H. We assume that the agent monotonicity condition is fulfilled, meaning that $MRS_{zx}^L > MRS_{zx}^H$ holds at any vector (z,x), where MRS_{zx}^i is defined as $MRS_{zx}^i \equiv -(\partial v_i/\partial z_i)/(\partial v_i/\partial x_i)$. This implies that for any income tax function the high-able individual does not choose to earn less income than the low-able.

The resource constraint reads

$$\pi_L(z_L - x_L) + \pi_H(z_H - x_H) + \sum_{j=1,2,3} \tau_j(\pi_L c_{jL} + \pi_H c_{jH}) \geq g, \tag{7}$$

i.e. tax revenues have to be raised to finance exogenous public spending g. In addition the government is constrained by a self-selection constraint, which is given by

$$v_H(x_H, z_H, p_1, p_2, p_3, \overline{c_{1H}}, \overline{c_{2H}}) \geq v_H(x_L, z_L, p_1, p_2, p_3, \overline{c_{1L}}, \overline{c_{2L}}). \tag{8}$$

It guarantees that the high-able individual does not prefer the bundle which is designed for the low-able individual. The constraint that the L type does not mimic the H type is not binding in the optimum and therefore neglected, given that we restrict the analysis to cases, where the government wants to redistribute from high- to low-ability persons. Observe that we assume that the reference levels for the mimicker and the L types are identical, i.e. type H when mimicking compares his/her consumption to $\overline{c_{kL}}$, $k = 1, 2$. Hence, we assume that income (and not ability) is decisive for the chosen reference level.[11] To abbreviate notation indirect utility of the mimicker is denoted by $v_H[L]$ and consumption of the mimicker by $c_{jH}[L]$.

In addition the definitions of the reference levels given by (1) and (2) are taken into account as separate (equality) constraints. The Lagrange multipliers for the resource and the self-selection constraint are denoted by λ and μ, respectively, and the multipliers for the reference levels are given by $\gamma_{1L}, \gamma_{1H}, \gamma_{2L}, \gamma_{2H}$. The Lagrangean function of the government's maximization problem can thus be written as

$$\begin{aligned} \mathcal{L} \;=\;& f_L v_L + f_H v_H + \lambda[\pi_L(z_L - x_L) + \pi_H(z_H - x_H) + \sum_{j=1,2,3} \tau_j(\pi_L c_{jL} + \pi_H c_{jH}) - g] \\ & +\mu(v_H - v_H[L]) + \gamma_{1L}(\overline{c_{1L}} - \alpha_{1L}(L)\pi_L c_{1L} - \alpha_{1H}(L)\pi_H c_{1H}) + \gamma_{1H}(\overline{c_{1H}} \\ & -\alpha_{1L}(H)\pi_L c_{1L} - \alpha_{1H}(H)\pi_H c_{1H}) + \gamma_{2L}(\overline{c_{2L}} - \alpha_{2L}(L)\pi_L c_{2L} - \alpha_{2H}(L)\pi_H c_{2H}) \\ & +\gamma_{2H}(\overline{c_{2H}} - \alpha_{2L}(H)\pi_L c_{2L} - \alpha_{2H}(H)\pi_H c_{2H}). \end{aligned} \tag{9}$$

[11] A similar assumption is made by Micheletto (2010). However, one could also think of a situation where the mimicker continues to compare his/her consumption to the reference levels of the H type. As our main results remain valid under both variants, we chose the one that seems more plausible to us.

The government maximizes the Lagrangean with respect to x_i, z_i, τ_j and also with respect to $\overline{c_{ki}}$. The first-order conditions for this maximization problem are provided in the Appendix. To summarize, the goal of the government is to design a tax system that redistributes income in an efficient way and that internalizes the externalities induced by the consumption of commodities 1 and 2.

3 Optimal taxation of commodities

As is well known from the literature (e.g. Pirttilä and Tuomala, 1997; Cremer et al., 1998), the result by Atkinson and Stiglitz (1976), which states that commodity taxes are redundant in the presence of an optimal nonlinear income tax if preferences are weakly separable in labor supply and consumption, does not hold if there are consumption externalities. Taxing the externality-generating good increases welfare as this allows to internalize the externality. Further, it has been shown that the additivity property discovered by Sandmo (1975) continues to hold in this more general model of the Mirrleesian type. The additivity property consists of two characteristics. First, the presence of an externality-generating good only alters the tax formula for that particular good and leaves other tax instruments unaffected by the externality. Second, the Pigouvian part of taxation appears additively in the tax formula. However, in a recent paper Micheletto (2008) has shown that for the 'additivity property' to hold in this context it is essential that the externality is of the 'atmospheric' type. If individuals of different ability are not equally effective as externality-generating units, then the 'additivity property' is in general violated and a role for other tax instruments arises to internalize the externality.

We generalize this discussion to a setting where there are two externality-generating goods, while the previous literature considered only one externality-generating good. This allows us to analyze the interdependence of the two externality-generating commodities and its consequences for the optimal tax structure. Further, we focus on positional externalities, where in contrast to the 'atmospheric' type it in fact matters which individual increases with his/her consumption the level of the externality (see equations 1 and 2).

3.1 Shadow prices

We start this section with a discussion of the shadow prices of the externalities, as they will be important for our discussion of optimal income and commodity taxes later on. In the derivation of the expressions for the shadow prices we extend the approach by Pirttilä and Tuomala (1997), which we generalize for our purposes. Let the shadow prices (measured in terms of the government's tax revenues) be denoted by γ_{ki}/λ, $k = 1, 2$ and $i = L, H$, and let the marginal rate of substitution between $\overline{c_{ki}}$ and x_i be defined by

$$MWP_{ki} \equiv -\frac{\partial v_i / \partial \overline{c_{ki}}}{\partial v_i / \partial x_i}. \tag{10}$$

It can be interpreted as the marginal willingness to pay of an individual i to reduce the reference consumption level $\overline{c_{ki}}$ by one unit. Lemma 1 describes the system of equations that determines the shadow prices of the externalities.

Lemma 1: In our model the shadow prices of the externalities measured in terms of tax revenues are determined by the following system of equations:

$$A \begin{pmatrix} \gamma_{1L}/\lambda \\ \gamma_{2L}/\lambda \\ \gamma_{1H}/\lambda \\ \gamma_{2H}/\lambda \end{pmatrix} = \begin{pmatrix} \pi_L MWP_{1L} - \frac{\mu}{\lambda} \frac{\partial v_H[L]}{\partial x_L} (MWP_{1H}[L] - MWP_{1L}) - \pi_L \sum_{j=1,2,3} \tau_j \frac{\partial c_{jL}^{com}}{\partial \overline{c_{1L}}} \\ \pi_L MWP_{2L} - \frac{\mu}{\lambda} \frac{\partial v_H[L]}{\partial x_L} (MWP_{2H}[L] - MWP_{2L}) - \pi_L \sum_{j=1,2,3} \tau_j \frac{\partial c_{jL}^{com}}{\partial \overline{c_{2L}}} \\ \pi_H MWP_{1H} - \pi_H \sum_{j=1,2,3} \tau_j \frac{\partial c_{jH}^{com}}{\partial \overline{c_{1H}}} \\ \pi_H MWP_{2H} - \pi_H \sum_{j=1,2,3} \tau_j \frac{\partial c_{jH}^{com}}{\partial \overline{c_{2H}}} \end{pmatrix}, \tag{11}$$

where compensated demand for commodity j of an individual i is denoted by c_{ji}^{com}. We call A a feedback matrix, which is given by

$$A = \begin{pmatrix} (1 - \alpha_{1L}(L)\pi_L \frac{\partial c_{1L}^{com}}{\partial \overline{c_{1L}}}) & -\alpha_{2L}(L)\pi_L \frac{\partial c_{2L}^{com}}{\partial \overline{c_{1L}}} & -\alpha_{1L}(H)\pi_L \frac{\partial c_{1L}^{com}}{\partial \overline{c_{1L}}} & -\alpha_{2L}(H)\pi_L \frac{\partial c_{2L}^{com}}{\partial \overline{c_{1L}}} \\ -\alpha_{1L}(L)\pi_L \frac{\partial c_{1L}^{com}}{\partial \overline{c_{2L}}} & (1 - \alpha_{2L}(L)\pi_L \frac{\partial c_{2L}^{com}}{\partial \overline{c_{2L}}}) & -\alpha_{1L}(H)\pi_L \frac{\partial c_{1L}^{com}}{\partial \overline{c_{2L}}} & -\alpha_{2L}(H)\pi_L \frac{\partial c_{2L}^{com}}{\partial \overline{c_{2L}}} \\ -\alpha_{1H}(L)\pi_H \frac{\partial c_{1H}^{com}}{\partial \overline{c_{1H}}} & -\alpha_{2H}(L)\pi_H \frac{\partial c_{2H}^{com}}{\partial \overline{c_{1H}}} & (1 - \alpha_{1H}(H)\pi_H \frac{\partial c_{1H}^{com}}{\partial \overline{c_{1H}}}) & -\alpha_{2H}(H)\pi_H \frac{\partial c_{2H}^{com}}{\partial \overline{c_{1H}}} \\ -\alpha_{1H}(L)\pi_H \frac{\partial c_{1H}^{com}}{\partial \overline{c_{2H}}} & -\alpha_{2H}(L)\pi_H \frac{\partial c_{2H}^{com}}{\partial \overline{c_{2H}}} & -\alpha_{1H}(H)\pi_H \frac{\partial c_{1H}^{com}}{\partial \overline{c_{2H}}} & (1 - \alpha_{2H}(H)\pi_H \frac{\partial c_{2H}^{com}}{\partial \overline{c_{2H}}}) \end{pmatrix}.$$

Proof: A derivation of Lemma 1 is provided in the Appendix.

As is shown in the Appendix, Lemma 1 follows from the first-order conditions of the government's maximization problem. Altogether there are four shadow prices, two for each type. They can be interpreted as the social harm or gain of a specific externality measured in terms of tax revenues.

Lemma 1 reveals some interesting implications of our multi-externality setting. The feedback matrix A is a generalization of the feedback parameter discussed by Pirttilä and Tuomala (1997) for the case where only the consumption of one good causes an externality and where both types face the same externality. It captures the weighted reaction of compensated demand for the externality-generating commodities 1 and 2 in case of a marginal increase of the level of a specific externality. If those effects are nonzero, i.e. if $\partial c_{1i}^{com}/\partial \overline{c_{2i}} \neq 0$ and $\partial c_{2i}^{com}/\partial \overline{c_{1i}} \neq 0$, then the shadow prices depend on each other. That is, the social harm or gain of a specific externality also depends on the social harm or gain of the other externalities.

On the right-hand side of (11) one can see that γ_{kL}/λ and γ_{kH}/λ, $k = 1, 2$, differ in one important aspect. The shadow prices of the H types contain no impact on the self-selection constraint. This is because of our assumption that a high-able individual when mimicking compares his/her consumption to the reference levels of the L types. The sign of the self-selection based part $\frac{\mu}{\lambda}\frac{\partial v_H[L]}{\partial x_L}(MWP_{kH}[L] - MWP_{kL})$ is ambiguous. It depends on whether the mimicker or the L-type has a higher marginal willingness to pay to avoid the externality. Note that the only difference between them is labor supply provided as the mimicker is more productive but their income is the same, i.e. the sign depends on $\partial MWP_{ki}/\partial l_i \gtreqless 0$. If the mimicker has a higher marginal willingness to pay to avoid the externality than the L type the value of the shadow price is reduced, because then the mimicker is hurt more by the externality and thus an increase of the reference consumption level gives slack to the self-selection constraint. Obviously the term is zero if the marginal willingness to pay to avoid the externality is independent of leisure. However, to our knowledge no empirical evidence on this issue exists, which is why the sign of the effect on the self-selection constraint is left open.

All four shadow prices depend on the direct harmful effect of an externality given by $\pi_i MWP_{ki}$. It is the total marginal willingness to pay of all individuals of type i to avoid an externality. As its sign is always positive it increases the value of the shadow price. In addition, the shadow prices depend on the effect of an externality on the government's tax revenues, which is given by $\pi_i \sum_{j=1,2,3} \tau_j \frac{\partial c_{ji}}{\partial \overline{c_{1i}}}$ and $\pi_i \sum_{j=1,2,3} \tau_j \frac{\partial c_{ji}}{\partial \overline{c_{2i}}}$, $i = L, H$, respectively. It describes by how much commodity tax revenues change if the reference consumption level of good k marginally increases. The sign of this term is again ambiguous, as the reaction of compensated demands due to a change of $\overline{c_{ki}}$ can have either sign.

In general, the shadow prices measured in terms of tax revenues can be either positive or negative, because the sign of some terms in (11) is ambiguous. That is, in principle an increase in the reference level can also lead to a social gain. However, the case that an increase of the reference consumption level is socially harmful (positive shadow prices) appears more plausible because of the direct negative effect.

3.2 Optimal commodity tax rates

In the Appendix we prove that optimal commodity tax rates have to satisfy

$$B\begin{pmatrix}\tau_1\\ \tau_2\\ \tau_3\end{pmatrix} = \begin{pmatrix} -\frac{\mu}{\lambda}\frac{\partial v_H[L]}{\partial x_L}(c_{1H}[L]-c_{1L}) + \sum_{k=1,2}\sum_i(\frac{\gamma_{kL}}{\lambda}\alpha_{ki}(L)\pi_i\frac{\partial c_{ki}^{com}}{\partial p_1} + \frac{\gamma_{kH}}{\lambda}\alpha_{ki}(H)\pi_i\frac{\partial c_{ki}^{com}}{\partial p_1})\\ -\frac{\mu}{\lambda}\frac{\partial v_H[L]}{\partial x_L}(c_{2H}[L]-c_{2L}) + \sum_{k=1,2}\sum_i(\frac{\gamma_{kL}}{\lambda}\alpha_{ki}(L)\pi_i\frac{\partial c_{ki}^{com}}{\partial p_2} + \frac{\gamma_{kH}}{\lambda}\alpha_{ki}(H)\pi_i\frac{\partial c_{ki}^{com}}{\partial p_2})\\ -\frac{\mu}{\lambda}\frac{\partial v_H[L]}{\partial x_L}(c_{3H}[L]-c_{3L}) + \sum_{k=1,2}\sum_i(\frac{\gamma_{kL}}{\lambda}\alpha_{ki}(L)\pi_i\frac{\partial c_{ki}^{com}}{\partial p_3} + \frac{\gamma_{kH}}{\lambda}\alpha_{ki}(H)\pi_i\frac{\partial c_{ki}^{com}}{\partial p_3})\end{pmatrix}, \quad (12)$$

where

$$B = \begin{pmatrix} \sum_i \pi_i\frac{\partial c_{1i}^{com}}{\partial p_1} & \sum_i \pi_i\frac{\partial c_{2i}^{com}}{\partial p_1} & \sum_i \pi_i\frac{\partial c_{3i}^{com}}{\partial p_1}\\ \sum_i \pi_i\frac{\partial c_{1i}^{com}}{\partial p_2} & \sum_i \pi_i\frac{\partial c_{2i}^{com}}{\partial p_2} & \sum_i \pi_i\frac{\partial c_{3i}^{com}}{\partial p_2}\\ \sum_i \pi_i\frac{\partial c_{1i}^{com}}{\partial p_3} & \sum_i \pi_i\frac{\partial c_{2i}^{com}}{\partial p_3} & \sum_i \pi_i\frac{\partial c_{3i}^{com}}{\partial p_3}\end{pmatrix}. \quad (13)$$

Equation (12) follows from the first-order conditions of the government's maximization problem. Applying Cramer's rule one can derive an implicit solution for τ_1, τ_2, τ_3. We start with a discussion of the optimal tax rate on the nonpositional good which is implicitly determined by

$$\tau_3 = \frac{1}{|B|}\psi_3 + \frac{\pi_L\pi_H}{|B|}\sum_{k=1,2}[\frac{\gamma_{kL}}{\lambda}(\alpha_{kH}(L)-\alpha_{kL}(L)) + \frac{\gamma_{kH}}{\lambda}(\alpha_{kH}(H)-\alpha_{1L}(H))]D_k, \quad (14)$$

where $|B|$ is the determinant of matrix B and where D_k is defined as

$$\begin{aligned} D_k \equiv\ & \sum_i \pi_i\frac{\partial c_{ni}^{com}}{\partial p_n}(\frac{\partial c_{kL}^{com}}{\partial p_k}\frac{\partial c_{kH}^{com}}{\partial p_3} - \frac{\partial c_{kH}^{com}}{\partial p_k}\frac{\partial c_{kL}^{com}}{\partial p_3}) + \sum_i \pi_i\frac{\partial c_{ni}^{com}}{\partial p_k}(\frac{\partial c_{kL}^{com}}{\partial p_3}\frac{\partial c_{kH}^{com}}{\partial p_n} - \frac{\partial c_{kH}^{com}}{\partial p_3}\frac{\partial c_{kL}^{com}}{\partial p_n})\\ & + \sum_i \pi_i\frac{\partial c_{ni}^{com}}{\partial p_3}(\frac{\partial c_{kL}^{com}}{\partial p_n}\frac{\partial c_{kH}^{com}}{\partial p_k} - \frac{\partial c_{kH}^{com}}{\partial p_n}\frac{\partial c_{kL}^{com}}{\partial p_k}), \end{aligned} \quad (15)$$

with $k, n = 1, 2$ and $k \neq n$. For expositional reasons we have introduced the variable ψ_j, $j = 1, 2, 3$, which captures the well-known self-selection part of commodity taxation. A formal definition of ψ_j is also provided in the Appendix. This self-selection part also appears in a model without externalities and can be considered as the non-Pigouvian part of the tax formula. It is zero if preferences are weakly separable in labor supply and consumption as then the mimicker and the L type have the same consumption pattern, that is $c_{jH}[L] = c_{jL}, j = 1, 2, 3$. In that case commodity taxes have no effect on the self-selection constraint (and are redundant in the absence of externalities).

The interesting aspect of equation (14) is, however, that in addition to the self-selection based part the shadow prices of the externalities appear in the tax formula. Thus, in our model it is in general optimal to also tax the nonpositional good for externality-correcting purposes, which violates the 'additivity property'. But one immediately observes that the Pigouvian part in (14) is zero if $\alpha_{kL}(i) = \alpha_{kH}(i)$, $k = 1, 2$ and $i = L, H$, i.e. if the consumption of the two types is weighted equally in the formation of the reference levels. Also if there are no compensated cross-price effects, the Pigouvian elements from (14) disappear as then $D_k = 0$. But even if cross-price effects are nonzero and $\alpha_{kL}(i) \neq \alpha_{kH}(i)$, $k = 1, 2$ and $i = L, H$, a condition exists under which the optimal tax rate on the nonpositional good remains unaffected by the externalities. If preferences are such that

$$\frac{\partial c_{kL}^{com}}{\partial p_j} / \frac{\partial c_{kL}^{com}}{\partial p_m} = \frac{\partial c_{kH}^{com}}{\partial p_j} / \frac{\partial c_{kH}^{com}}{\partial p_m}, \tag{16}$$

$k = 1, 2$ and $j, m = 1, 2, 3$, the Pigouvian parts from (14) are zero even if cross-price effects are present and even if the consumption of the two types is weighted differently in the formation of the reference levels. This follows from the fact that D_k is equal to zero if (16) holds. The condition given by (16) is illuminating as it highlights the role that a tax on the nonpositional good can play to combat the externalities induced by the consumption of the positional commodities. If the proportions of compensated price effects (own to cross and cross to cross price effects) for commodities 1 and 2 are the same for both types as stated in (16), then commodity 3 should not be taxed to correct for the externalities. In other words, if the two types react in the same way to relative price changes, then by taxing the nonpositional good no additional targeting can be achieved, which would be desirable if the consumption of the two types were weighted differently. But if, loosely speaking, introducing a tax on the nonpositional commodity modifies the consumption baskets of the two types such that the type, whose consumption is weighted stronger in the formation of the reference levels, consumes relatively more of the nonpositional good than the other type after the introduction of the tax, then τ_3 serves to correct for the externalities. Proposition 1 summarizes this result.

Proposition 1: In general a tax on the nonpositional commodity serves to correct for the externalities induced by the consumption of commodities 1 and 2 violating the 'additivity property'. If, however, at least one of the following properties holds, then τ_3 contains no Pigouvian elements and is zero given that preferences are weakly separable in labor supply and consumption. These properties are

(i) equal weights $\alpha_{kL}(i) = \alpha_{kH}(i)$, $k = 1, 2$ and $i = L, H$,

(ii) no cross-price effects,

(iii) equal proportions of compensated price effects as given by (16).

The intuition for this result is that if the two types are equally effective as externality-generating units then the linearity restriction of τ_1 and τ_2 does not matter for externality-correcting purposes. Otherwise additional differentiation between them is desirable, which is possible with the full set of commodity taxes if cross-price effects are present and if the two types react differently to changes in τ_1, τ_2, τ_3.

Next we turn to the optimal tax rates on commodities 1 and 2. They are implicitly given by

$$\begin{aligned} \tau_k \quad &= \quad \frac{1}{|B|}\psi_k + \frac{1}{|B|}(\frac{\gamma_{kL}}{\lambda}E_{kL} + \frac{\gamma_{kH}}{\lambda}E_{kH}) \\ &+\frac{\pi_L\pi_H}{|B|}[\frac{\gamma_{nL}}{\lambda}(\alpha_{nH}(L) - \alpha_{nL}(L)) + \frac{\gamma_{nH}}{\lambda}(\alpha_{nH}(H) - \alpha_{nL}(H))]F_k, \end{aligned} \tag{17}$$

with $k, n = 1, 2$ and $k \neq n$, and where E_{ks}, $s = L, H$, and F_k are defined by

$$\begin{aligned} E_{ks} \quad \equiv \quad & \sum_i \alpha_{ki}(s)\pi_i \frac{\partial c_{ki}^{com}}{\partial p_k}(\sum_i \pi_i \frac{\partial c_{ni}^{com}}{\partial p_n}\sum_i \pi_i \frac{\partial c_{3i}^{com}}{\partial p_3} - \sum_i \pi_i \frac{\partial c_{ni}^{com}}{\partial p_3}\sum_i \pi_i \frac{\partial c_{3i}^{com}}{\partial p_n}) \\ &+\sum_i \alpha_{ki}(s)\pi_i \frac{\partial c_{ki}^{com}}{\partial p_n}(\sum_i \pi_i \frac{\partial c_{3i}^{com}}{\partial p_k}\sum_i \pi_i \frac{\partial c_{ni}^{com}}{\partial p_3} - \sum_i \pi_i \frac{\partial c_{ni}^{com}}{\partial p_k}\sum_i \pi_i \frac{\partial c_{3i}^{com}}{\partial p_3}) \\ &+\sum_i \alpha_{ki}(s)\pi_i \frac{\partial c_{ki}^{com}}{\partial p_3}(\sum_i \pi_i \frac{\partial c_{ni}^{com}}{\partial p_k}\sum_i \pi_i \frac{\partial c_{3i}^{com}}{\partial p_n} - \sum_i \pi_i \frac{\partial c_{ni}^{com}}{\partial p_n}\sum_i \pi_i \frac{\partial c_{3i}^{com}}{\partial p_k}), \end{aligned} \tag{18}$$

$$\begin{aligned} F_k \quad \equiv \quad & \sum_i \pi_i \frac{\partial c_{3i}^{com}}{\partial p_3}(\frac{\partial c_{nL}^{com}}{\partial p_n}\frac{\partial c_{nH}^{com}}{\partial p_k} - \frac{\partial c_{nH}^{com}}{\partial p_n}\frac{\partial c_{nL}^{com}}{\partial p_k}) + \sum_i \pi_i \frac{\partial c_{3i}^{com}}{\partial p_n}(\frac{\partial c_{nL}^{com}}{\partial p_k}\frac{\partial c_{nH}^{com}}{\partial p_3} - \frac{\partial c_{nH}^{com}}{\partial p_k}\frac{\partial c_{nL}^{com}}{\partial p_3}) \\ &+\sum_i \pi_i \frac{\partial c_{3i}^{com}}{\partial p_k}(\frac{\partial c_{nL}^{com}}{\partial p_3}\frac{\partial c_{nH}^{com}}{\partial p_n} - \frac{\partial c_{nH}^{com}}{\partial p_3}\frac{\partial c_{nL}^{com}}{\partial p_n}). \end{aligned} \tag{19}$$

One can see that also the tax rates on the positional commodities depend in a complex way on all four shadow prices. Remarkably, also the shadow prices of the respective other externality-generating commodity appear explicitly in the optimal tax formula, i.e. τ_1 and τ_2 depend on both externalities. This is also in contradiction to the 'additivity property' which states that the presence of an externality only alters the tax formula on that particular good.

To get a better intuition for equation (17) assume for a moment equal weights $\alpha_{kL}(i) = \alpha_{kH}(i) = \alpha_k(i)$, $k = 1, 2$ and $i = L, H$.[12] Then (17) reduces to

$$\tau_k = \frac{1}{|B|}\psi_k + \frac{\gamma_{kL}}{\lambda}\alpha_k(L) + \frac{\gamma_{kH}}{\lambda}\alpha_k(H), \tag{20}$$

[12]This assumption guarantees, loosely speaking, that it does not matter who consumes a positional commodity. But the two types still might have different reference levels to which they compare their consumption to, as $\alpha_k(L) \neq \alpha_k(H)$ is still possible.

$k = 1, 2$. Observe that with equal weights the definition given in (18) reduces to $E_{ks} \equiv \alpha_k(s) |B|$, $s = L, H$. Further, the effect of the shadow prices of the other externality-generating good given by the second line on the RHS of (17) is zero in that case. The optimal tax formula given by equation (20) looks similar to those provided by Pirttilä and Tuomala (1997) and Cremer et al. (1998) where the externality is assumed to be of the 'atmospheric' type. The main difference is that even with equal weights the high- and the low-able still might face different externalities in our framework, and hence the Pigouvian part in (20) consists of a weighted sum of the shadow prices for the two income groups.

The most striking difference between (17) and (20) is that in (20) only the shadow prices of the taxed commodity appear explicitly in the optimal tax formula.[13] Thus, the fact that for positional externalities it might matter who consumes the status good has strong implications for the optimal commodity tax structure. But also if $\alpha_{kL}(i) \neq \alpha_{kH}(i)$, $k = 1, 2$, the part that includes the shadow prices of the other externality-generating commodity (second line in (17)) cancels out if there are no cross-price effects and/or the proportions in equation (16) hold as then F_k given by (19) is equal to zero. This is of course related to the result provided in Proposition 1. Also the intuition is similar. If the consumption of the two types is weighted differently in the formation of the reference levels, then additional differentiation between them is desirable for externality-correcting purposes. This additional differentiation is possible with the set of commodity taxes if the two types react differently to relative price changes and if cross-price effects are present. The potential role of the income tax in order to achieve this additional differentiation will be discussed in section 4. Proposition 2 summarizes the result for the optimal tax rates on the positional commodities 1 and 2.

Proposition 2: In general the optimal tax rates on the positional commodities depend explicitly on the shadow prices of both externalities which violates the 'additivity property'. But if one of the following properties applies, γ_{ni}/λ affects τ_k only indirectly through its effect on γ_{ki}/λ (see Lemma 1), with $k, n = 1, 2$ and $k \neq n$. These properties are:

(i) $\alpha_{kH}(i) = \alpha_{kL}(i)$, $i = L, H$,

(ii) no cross-price effects,

(iii) equal proportions of compensated price effects as given by (16).

[13] Note, however, that from Lemma 1 we know that as a consequence of our multi-externality assumption the shadow prices depend on each other if $\partial c_{1i}^{com}/\partial \overline{c_{2i}} \neq 0$ and $\partial c_{2i}^{com}/\partial \overline{c_{1i}} \neq 0$. Then, even with equal weights, the tax rates on the positional commodities are affected by all shadow prices due to this interdependence of the externalities.

Altogether from Propositions 1 and 2 we can conclude that the 'additivity property', stating that an externality is best addressed by imposing a tax directly on the externality-generating commodity and leaving all other tax instruments unaffected by the externality, does not hold in our model. Rather the full set of commodity taxes should be applied to achieve an efficient allocation by internalizing the externalities. Concerning the sign of optimal commodity tax rates not much can be said in particular with respect to τ_3. Also the tax rates on the the positional commodities can in principle have either sign, but if shadow prices are positive τ_1 and τ_2 are likely to be positive. In order to be able to make a precise statement about the sign and the magnitude of optimal commodity tax rates one would have to specify preferences and make specific assumptions about the reference levels.

Two examples: upward comparison and within-group comparison

Two possible variants of our model that are frequently discussed in the literature are upward and withín-group comparison. Both of them can easily be implemented into our model just by adapting weights properly. Hence, all results from above can be applied to these special cases.

In particular upward comparison received a lot of attention from both the theoretical and empirical literature.[14] There are two alternative approaches to implement the idea of upward comparison into our model. In the first approach all individuals compare their consumption to the consumption of the high-able individuals, while in the second one only the L types compare their consumption to the one of the H types and the H-types themselfes have no positional concerns. For the former the reference levels are then given by $\overline{c_{kL}} = \alpha_{kH}(L)\pi_H c_{kH}$ and $\overline{c_{kH}} = \alpha_{kH}(H)\pi_H c_{kH}$, $k = 1, 2$. For the latter again $\overline{c_{kL}} = \alpha_{kH}(L)\pi_H c_{kH}$, $k = 1, 2$, but the H types make no comparisons in that case. In both cases the two types are not equally effective as externality-generating units as only the consumption of the H types increases the reference levels.[15]

Assuming within-group comparisons, means that individuals only compare themselves with individuals in the same income group.[16] Indeed, some evidence suggests that people mainly compare themselves with similar individuals (see for example Runciman, 1966). The reference levels are then equal to $\overline{c_{kH}} = \alpha_{kH}(H)\pi_H c_{kH}$ and $\overline{c_{kL}} = \alpha_{kL}(L)\pi_L c_{kL}$. From this formulation it follows that the consumption of an individual i only imposes an externality on other individuals with the same income.

[14] See for example Micheletto (2010) and Aronsson and Johansson-Stenman (2010) for theoretical studies assuming upward comparison and Bowles and Park (2005) for an empirical study supporting this assumption.

[15] The weights $\alpha_{kL}(i)$ are set equal to zero in both cases, implying that the consumption of the L types does not cause an externality.

[16] See for example Knell (1999) for an application of such a reference level.

It follows from Proposition 1 and 2 that in both cases, it is in general optimal to use all available commodity taxes to internalize the externalities induced by the consumption of commodities 1 and 2. Clearly, for this statement to be true it is essential that cross-price effects are present and that equation (16) does not hold.

4 Marginal income tax rates

In the previous section we have established the result that it is in general optimal to use the whole set of commodity taxes, notably also the tax on the nonpositional good, in order to correct for the externalities. In this section we analyze the potential role of the income tax to serve for the same purpose, that is, we want to find out if the presence of the externalities affects the optimal income tax schedule, and if yes, how.

The marginal income tax rates can easily be obtained by dividing the first-order conditions for z_i and x_i (A3 by A1 and A4 by A2) and by using equation (5). The intuition is that the government chooses the optimal gross and net income for each individual and then determines an income tax function such that individuals realize those bundles. In the Appendix we show that the optimal marginal income tax rates for arbitrary commodity taxes are given by

$$\begin{aligned} T'(z_H) &= \sum_{k=1,2}(\frac{\gamma_{kL}}{\lambda}\alpha_{kH}(L) + \frac{\gamma_{kH}}{\lambda}\alpha_{kH}(H) - \tau_k)(\frac{\partial c_{kH}}{\partial z_H} + MRS^H_{zx}\frac{\partial c_{kH}}{\partial x_H}) \\ &\quad -\tau_3(\frac{\partial c_{3H}}{\partial z_H} + MRS^H_{zx}\frac{\partial c_{3H}}{\partial x_H}), \end{aligned} \tag{21}$$

$$\begin{aligned} T'(z_L) &= \sum_{k=1,2}(\frac{\gamma_{kL}}{\lambda}\alpha_{kL}(L) + \frac{\gamma_{kH}}{\lambda}\alpha_{kL}(H) - \tau_k)(\frac{\partial c_{kL}}{\partial z_L} + MRS^L_{zx}\frac{\partial c_{kL}}{\partial x_L}) \\ &\quad -\tau_3(\frac{\partial c_{3L}}{\partial z_L} + MRS^L_{zx}\frac{\partial c_{3L}}{\partial x_L}) + \frac{\mu}{\lambda\pi_L}\frac{\partial v_H[L]}{\partial x_L}(MRS^L_{zx} - MRS^H_{zx}[L]). \end{aligned} \tag{22}$$

From (21) and (22) it can be concluded that in general also the marginal income tax rates are affected by the externalities, which implies that the 'additivity property' is also violated with respect to the income tax. This can immediately be seen by plugging in the optimal commodity tax rates given by (14) and (17) into (21) and (22). Then one observes that the optimal income tax schedule depends explicitly on all shadow prices of the externalities. If, however, $\alpha_{kH}(i) = \alpha_{kL}(i)$, $k = 1, 2$ and $i = L, H$ (property (i) in Proposition 1), then the 'additivity property' is also restored with respect to the income tax as then the Pigouvian elements (the sum term) in (21) and (22) cancel out. Note that with equal weights optimal commodity tax rates on goods 1 and 2 are given by (20) and the tax rate on the nonpositional good reduces to $\tau_3 = \frac{1}{|B|}\psi_3$, from which it follows that the Pigouvian parts in (21) and (22) drop out.

An interesting difference between the results concerning the income tax and the tax on the nonpositional good is that from Proposition 1 we know that even if the two types are not equally effective as externality-generating units, τ_3 is unaffected by the externalities if there are no cross price effects and/or equation (16) holds (properties (ii) and (iii)). However, the same is not true for the marginal income tax rates which are influenced by the externalities as soon as $\alpha_{kH}(i) \neq \alpha_{kL}(i)$. A closely related result is due to Micheletto (2008) who showed that whenever the 'additivity property' is violated with respect to the commodity tax structure, also the income tax schedule contains Pigouvian elements, but that the reverse is not necessarily true. We have derived explicit conditions for this result. Proposition 3 summarizes our characterization of the income tax schedule.

Proposition 3: In general the optimal marginal income tax rates for both types depend on the externalities. Thus, the 'additivity property' is also violated with respect to the income tax. Only if $\alpha_{kH}(i) = \alpha_{kL}(i)$, $k = 1, 2$ and $i = L, H$, the positional externalities have no impact on the income tax.

Note that without further assumptions the sign of the Pigouvian parts in the income tax schedule is ambiguous, even if shadow prices are assumed to be positive. It depends on whether the optimal commodity tax rates τ_k, $k = 1, 2$ are larger or smaller than the weighted sum of the shadow prices of the taxed commodity k, on whether the status goods are complements or substitutes to leisure ($\partial c_{ki}/\partial z_i \gtreqless 0$) and on whether demand for the status goods increases with net income ($\partial c_{ki}/\partial x_i \gtreqless 0$). In the next subsection we discuss the optimal income tax schedule if commodity taxes are restricted to be uniform. For this scenario more precise statements concerning the sign and the shape of the Pigouvian parts in the marginal income tax rates can be made.

4.1 Optimal income tax schedule with uniform taxation of commodities

One potential objection against our results is that differential taxation of status goods might not be feasible for political economy reasons.[17] In this subsection we touch upon this issue and assume that total consumption is restricted to be taxed uniformly at a rate τ, i.e. differential taxation of status goods is no longer possible. Without loss of generality we set $\tau = 0$ since the effect of any uniform consumption tax can also be attained through the income tax. Given this restriction on

[17] This point has been raised for example by Ireland (2001) and Frank (1999, 2008).

the commodity tax structure we analyze the role of the income tax to internalize the positional externalities. The optimal marginal income tax rates can again be obtained by combining the first-order-conditions for x_i, z_i with equation (5) and by taking into account the restriction that $\tau = 0$. From (21) and (22) it follows that in this case the optimal marginal income tax rates are given by

$$T'(z_H) = \sum_{k=1,2} (\frac{\gamma_{kL}}{\lambda} \alpha_{kH}(L) + \frac{\gamma_{kH}}{\lambda} \alpha_{kH}(H))(\frac{\partial c_{kH}}{\partial z_H} + MRS^H_{zx} \frac{\partial c_{kH}}{\partial x_H}), \tag{23}$$

$$\begin{aligned} T'(z_L) &= \sum_{k=1,2} (\frac{\gamma_{kL}}{\lambda} \alpha_{kL}(L) + \frac{\gamma_{kH}}{\lambda} \alpha_{kL}(H))(\frac{\partial c_{kL}}{\partial z_L} + MRS^L_{zx} \frac{\partial c_{kL}}{\partial x_L}) \\ &\quad + \frac{\mu}{\lambda \pi_L} \frac{\partial v_H[L]}{\partial x_L} (MRS^L_{zx} - MRS^H_{zx}[L]). \end{aligned} \tag{24}$$

In the abscence of commodity taxes only the income tax can be used to correct for the positional externalities. Hence, the income tax is affected by the externalities no matter how the reference level looks like. But surprisingly, the sign of the corrective parts in (23) and (24) (sum term) is still ambiguous, even if shadow prices are positive. This ambiguity in the sign of the Pigouvian part in the income tax schedule is an important difference to the results in previous studies on the optimal income tax when relative consumption matters, which assume that there is only one consumption good (see for example Aronsson and Johansson-Stenman, 2008). There, as soon as the shadow price is positive, the Pigouvian part in the income tax schedule is also positive.[18] Things become more complicated if there are both positional and nonpositional forms of consumption. For example, if a status good is a complement to leisure, and hence $\partial c_{ki}/\partial z_i < 0$, then there is an effect that works in the opposite direction requiring a lower or even negative marginal income tax rate. The intuition for this effect is that increasing the marginal income tax rate induces individuals to enjoy more leisure and to also consume more of the status good. Clearly, if the status good is a normal good, and hence $\partial c_{ki}/\partial x_i > 0$, this effect might be offset by the effect of a reduction in net income on the demand for the status good. The point is that if individuals can spend their income on positional and nonpositional goods, a higher income tax does not necessarily imply lower consumption of the positional good, which is in contrast to a model where all consumption is assumed to be positional. This result is stated in Proposition 4.

[18] Their formulas for the marginal income tax rates do not include a shadow price. In their notation, a sufficient condition for the relative consumption concerns to contribute to higher marginal income tax rates is that the low-ability type is at least as positional as the mimicker. In the absence of commodity taxes this condition would imply positive shadow prices in our framework, which justifies this statement.

Proposition 4: Assume that shadow prices are positive and that commodities 1 and 2 are normal goods, i.e. $\gamma_{ki}/\lambda > 0$ and $\partial c_{ki}/\partial x_i > 0$ with $k = 1, 2$ and $i = L, H$. Then the Pigouvian parts in the income tax schedule are unambiguously positive if demand for commodities 1 and 2 is either unaffected by leisure or decreases with leisure ($\partial c_{ki}/\partial z_i \geq 0$). If, however, the demand for status goods increases with leisure ($\partial c_{ki}/\partial z_i < 0$) the sign of the Pigouvian part is ambiguous.

Finally, we show that with some additional assumptions the Pigouvian part in the income tax schedule is progressive. The Pigouvian elements in (23) and (24) are given by the first term on the RHS of these equations. Assuming that the consumption of the H type has a larger weight in the formation of the reference levels (e.g. upward comparison), i.e. $\alpha_{kH}(i) > \alpha_{kL}(i)$, the inequality

$$\sum_{k=1,2}(\frac{\gamma_{kL}}{\lambda}\alpha_{kH}(L) + \frac{\gamma_{kH}}{\lambda}\alpha_{kH}(H)) > \sum_{k=1,2}(\frac{\gamma_{kL}}{\lambda}\alpha_{kL}(L) + \frac{\gamma_{kH}}{\lambda}\alpha_{kL}(H)) \tag{25}$$

holds, provided that the shadow prices are positive. Given $\alpha_{kH}(i) > \alpha_{kL}(i)$ a sufficient condition for the Pigouvian element in the optimal marginal income tax rate to be larger for the H type (see (23) and (24)), and hence, for the Pigouvian parts in the income tax schedule to be progressive, is

$$\underbrace{(\frac{\partial c_{kH}}{\partial z_H} + MRS^H_{zx}\frac{\partial c_{kH}}{\partial x_H})}_{(\frac{dc_{kH}}{dz_H})_{dV_H=0}} \geq \underbrace{(\frac{\partial c_{kL}}{\partial z_L} + MRS^L_{zx}\frac{\partial c_{kL}}{\partial x_L})}_{(\frac{dc_{kL}}{dz_L})_{dV_L=0}} > 0, \tag{26}$$

$k = 1, 2$. This condition states that the H types change their demand for the positional commodities at least as much as the L types, in case of a marginal increase of gross income which is compensated such that their utility does not change. Further, this change in demand has to be positive, which is guaranteed if commodity k is a normal good and if demand for k is unaffected by leisure or a substitute to leisure. Observe that if preferences are weakly separable in labor supply and consumption and homothetic in consumption this condition reduces to $MRS^H_{zx} \geq MRS^L_{zx}$ at the optimal bundles (z_H, x_H), (z_L, x_L) as with such preferences $\partial c_{ki}/\partial z_i = 0$ and $\partial c_{kH}/\partial x_H = \partial c_{kL}/\partial x_L$. In the absence of externalities the inequality $MRS^H_{zx} \geq MRS^L_{zx}$ holds at the optimal allocation, which is an immediate consequence of the zero at the top result in the standard version of the Mirrlees income tax model (Sadka, 1976; Seade, 1977). But in our model this inequality can in principle be violated as it is optimal to also distort the decision of the H types. Nevertheless, if the social damage caused by the externality is small, this condition is likely to hold. The result concerning the progressivity of the Pigouvian part in the income tax schedule is summarized in Proposition 5.

Proposition 5: Assume that $\alpha_{kH}(i) > \alpha_{kL}(i)$, $k = 1, 2$ and $i = L, H$. Then, if the externality is socially harmful (positive shadow prices), equation (26) is a sufficient but not necessary condition for the Pigouvian parts in the optimal income tax schedule to be progressive.

Thus, if the consumption of the H type has a larger weight in the formation of the externality, then it is optimal for the Pigouvian element in the income tax schedule to be larger for the H type, at least if the consumption of commodities 1 and 2 of the high-ables responds at least as strong to a change in gross-income than the one of the low-able.

5 Conclusion

In this paper we have studied the optimal taxation of income and commodities when individuals make relative consumption comparisons. In contrast to previous studies we have assumed that there are both positional and nonpositional commodities, taking into account the idea that relative consumption matters for some but not for all commodities. This view is also supported by recent results from the empirical literature (see for example Solnick and Hemenway, 2005). Moreover we have extended the literature on optimal taxation in the presence of externalities to a multi-externality setting which allows us to analyze the interdependence between the externality-generating commodities and its effect on the optimal tax structure.

We have found that in general the whole tax system is affected by the positional externalities. In particular even the tax rate on the nonpositional good and the income tax serve for externality-correcting purposes at least if the consumption of the two types is weighted differently in the formation of the reference levels (e.g. upward comparison). The reason is that in this case the proportional taxes on the positional commodities alone cannot achieve the additional differentiation between the two types which then becomes desirable for externality-correcting purposes. Some additional differentiation can be attained with the whole set of commodity taxes provided that cross-price effects are present and that the two types react differently to relative price changes. Further differentiation can be achieved with the income tax as it allows to tax the two types differently.

We have also analyzed the optimal tax structure if commodity taxes are restricted to be uniform for example due to political economy reasons. Then only the income tax can serve to correct for the positional externalities. In general the optimal marginal income tax rates are higher than in the absence of relative concerns and the well-known zero at the top result does not hold. However, our assumption that individuals can spend their income on both positional and nonpositional forms of consumption implies that if status consumption is a complement to leisure then the sign of the Pigouvian part in the income tax is ambiguous. The reason is that in this case an increase in the marginal income tax rate does not necessarily lead to a decline in the demand for status goods. Moreover we have shown that if the consumption of the high-able individuals has a higher weight in the formation of the reference levels then with some mild additional assumptions the Pigouvian elements in the marginal income tax rates are progressive.

Appendix

First order conditions of the government's maximization problem

The first-order conditions with respect to the income bundles x_i, z_i, $i = L, H$, are given by equations (A1)-(A4).

$$f_L \frac{v_L}{\partial x_L} = \lambda\pi_L - \lambda\pi_L \sum_{j=1,2,3} \tau_j \frac{\partial c_{jL}}{\partial x_L} + \mu \frac{\partial v_H[L]}{\partial x_L} + \pi_L \frac{\partial c_{1L}}{\partial x_L} \sum_{i=L,H} \gamma_{1i}\alpha_{1L}(i)$$
$$+ \pi_L \frac{\partial c_{2L}}{\partial x_L} \sum_{i=L,H} \gamma_{2i}\alpha_{2L}(i) \tag{A1}$$

$$f_H \frac{v_H}{\partial x_H} = \lambda\pi_H - \lambda\pi_H \sum_{j=1,2,3} \tau_j \frac{\partial c_{jH}}{\partial x_H} - \mu \frac{\partial v_H}{\partial x_H} + \pi_H \frac{\partial c_{1H}}{\partial x_H} \sum_{i=L,H} \gamma_{1i}\alpha_{1H}(i)$$
$$+ \pi_H \frac{\partial c_{2H}}{\partial x_H} \sum_{i=L,H} \gamma_{2i}\alpha_{2H}(i) \tag{A2}$$

$$f_L \frac{\partial v_L}{\partial z_L} = -\lambda\pi_L - \lambda\pi_L \sum_{j=1,2,3} \tau_j \frac{\partial c_{jL}}{\partial z_L} + \mu \frac{\partial v_H[L]}{\partial z_L} + \pi_L \frac{\partial c_{1L}}{\partial z_L} \sum_{i=L,H} \gamma_{1i}\alpha_{1L}(i)$$
$$+ \pi_L \frac{\partial c_{2L}}{\partial z_L} \sum_{i=L,H} \gamma_{2i}\alpha_{2L}(i) \tag{A3}$$

$$f_H \frac{\partial v_H}{\partial z_H} = -\lambda\pi_H - \lambda\pi_H \sum_{j=1,2,3} \tau_j \frac{\partial c_{jH}}{\partial z_H} - \mu \frac{\partial v_H}{\partial z_H} + \pi_H \frac{\partial c_{1H}}{\partial z_H} \sum_{i=L,H} \gamma_{1i}\alpha_{1H}(i)$$
$$+ \pi_H \frac{\partial c_{2H}}{\partial z_H} \sum_{i=L,H} \gamma_{2i}\alpha_{2H}(i) \tag{A4}$$

The first-order condition with respect to commodity taxes τ_j, $j = 1, 2, 3$, read

$$\sum_{i=L,H} f_i \frac{v_i}{\tau_j} + \lambda \sum_{i=L,H} \pi_i c_{ji} + \lambda \sum_{i=L,H} \sum_{m=1,2,3} \pi_i \tau_m \frac{\partial c_{mi}}{\partial \tau_j} + \mu(\frac{\partial v_H}{\partial \tau_j} - \frac{\partial v_H[L]}{\partial \tau_j})$$
$$- \gamma_{1L} \sum_{i=L,H} \alpha_{1i}(L)\pi_i \frac{\partial c_{1i}}{\partial \tau_j} - \gamma_{1H} \sum_{i=L,H} \alpha_{1i}(H)\pi_i \frac{\partial c_{1i}}{\partial \tau_j} - \gamma_{2L} \sum_{i=L,H} \alpha_{2i}(L)\pi_i \frac{\partial c_{2i}}{\partial \tau_j}$$
$$- \gamma_{2H} \sum_{i=L,H} \alpha_{2i}(H)\pi_i \frac{\partial c_{2i}}{\partial \tau_j} = 0. \tag{A5}$$

Finally, first order conditions for $\overline{c_{kL}}$ and $\overline{c_{kH}}$, $k = 1, 2$, are given by (A6)and (A7).

$$f_L \frac{\partial v_L}{\partial \overline{c_{kL}}} + \lambda \pi_L \sum_{j=1,2,3} \tau_j \frac{\partial c_{jL}}{\partial \overline{c_{kL}}} - \mu \frac{\partial v_H[L]}{\partial \overline{c_{kL}}} + \gamma_{kL} - \pi_L \frac{\partial c_{1L}}{\partial \overline{c_{kL}}} \sum_{i=L,H} \gamma_{1i}\alpha_{1L}(i)$$
$$- \pi_L \frac{\partial c_{2L}}{\partial \overline{c_{kL}}} \sum_{i=L,H} \gamma_{2i}\alpha_{2L}(i) = 0 \tag{A6}$$

$$f_H \frac{\partial v_H}{\partial \overline{c_{kH}}} + \lambda \pi_H \sum_{j=1,2,3} \tau_j \frac{\partial c_{jH}}{\partial \overline{c_{kH}}} + \mu \frac{\partial v_H}{\partial \overline{c_{kH}}} + \gamma_{kH} - \pi_H \frac{\partial c_{1H}}{\partial \overline{c_{kH}}} \sum_{i=L,H} \gamma_{1i}\alpha_{1H}(i)$$
$$- \pi_H \frac{\partial c_{2H}}{\partial \overline{c_{kH}}} \sum_{i=L,H} \gamma_{2i}\alpha_{2H}(i) = 0 \tag{A7}$$

Derivation of Lemma 1

Take the first order condition for $\overline{c_{kL}}$ given by A6 and add and subtract $\mu \frac{\partial v_H[L]}{\partial x_L} \frac{\partial v_L/\partial \overline{c_{kL}}}{\partial v_L/\partial x_L}$. A6 can then be written as

$$(f_L \frac{\partial v_L}{\partial x_L} - \mu \frac{\partial v_H[L]}{\partial x_L}) \frac{\partial v_L/\partial \overline{c_{kL}}}{\partial v_L/\partial x_L} - \mu \frac{\partial v_H[L]}{\partial x_L} (\frac{\partial v_H[L]/\partial \overline{c_{kL}}}{\partial v_H[L]/\partial x_L} - \frac{\partial v_L/\partial \overline{c_{kL}}}{\partial v_L/\partial x_L}) + \lambda \pi_L \sum_{j=1,2,3} \tau_j \frac{\partial c_{jL}}{\overline{c_{kL}}}$$
$$+ \gamma_{kL} - \pi_L \frac{\partial c_{1L}}{\partial \overline{c_{kL}}} \sum_i \gamma_{1i}\alpha_{1L}(i) - \pi_L \frac{\partial c_{2L}}{\partial \overline{c_{kL}}} \sum_i \gamma_{2i}\alpha_{2L}(i) = 0, \tag{A8}$$

$k = 1, 2$. Now make use of the definition for MWP_{ki} (equation (10)) and substitute for $(f_L \frac{\partial v_L}{\partial x_L} - \mu \frac{\partial v_H[L]}{\partial x_L})$ from the first-order condition for x_L given by A1. In addition use the Slutsky decompositions $\frac{\partial c_{ji}}{\partial \overline{c_{ki}}} = \frac{\partial c_{ji}^{com}}{\partial \overline{c_{ki}}} - MWP_{ki}\frac{\partial c_{ji}}{\partial x_i}$, $j = 1, 2, 3$ and $k = 1, 2$. Then one gets

$$\frac{\gamma_{kL}}{\lambda} - \frac{\gamma_{1L}}{\lambda}\alpha_{1L}(L)\pi_L\frac{\partial c_{1L}^{com}}{\partial \overline{c_{kL}}} - \frac{\gamma_{2L}}{\lambda}\alpha_{2L}(L)\pi_L\frac{\partial c_{2L}^{com}}{\partial \overline{c_{kL}}} - \frac{\gamma_{1H}}{\lambda}\alpha_{1L}(H)\pi_L\frac{\partial c_{1L}^{com}}{\partial \overline{c_{kL}}}$$
$$- \frac{\gamma_{2H}}{\lambda}\alpha_{2L}(H)\pi_L\frac{\partial c_{2L}^{com}}{\partial \overline{c_{kL}}} = \pi_L MWP_{kL} - \mu\frac{\partial v_H[L]}{\partial x_L}(MWP_{kH}[L] - MWP_{kL})$$
$$- \pi_L \sum_{j=1,2,3} \tau_j \frac{\partial c_{jL}^{com}}{\partial \overline{c_{kL}}}, \tag{A9}$$

k=1,2.

Similarly, the first-order condition for $\overline{c_{kH}}$ (equation A7) can also be written as

$$(f_H\frac{\partial v_H}{\partial x_H} + \mu\frac{\partial v_H}{\partial x_H})\frac{\partial v_H/\partial\overline{c_{kH}}}{\partial v_H/\partial x_H} + \lambda\pi_H\sum_{j=1,2,3}\tau_j\frac{\partial c_{jH}}{\partial\overline{c_{kH}}} + \gamma_{kH} - \pi_H\frac{\partial c_{1H}}{\partial\overline{c_{kH}}}\sum_i\gamma_{1i}\alpha_{1H}(i)$$
$$- \pi_H\frac{\partial c_{2H}}{\partial\overline{c_{kH}}}\sum_i\gamma_{2i}\alpha_{2H}(i) = 0, \tag{A10}$$

$k = 1, 2$. Substituting for $(f_H\frac{\partial v_H}{\partial x_H} + \mu\frac{\partial v_H}{\partial x_H})$ from A2 and again making use of the Slutsky decompositions, A10 becomes

$$\frac{\gamma_{kH}}{\lambda} - \frac{\gamma_{1L}}{\lambda}\alpha_{1H}(L)\pi_H\frac{\partial c_{1H}^{com}}{\partial \overline{c_{kH}}} - \frac{\gamma_{2L}}{\lambda}\alpha_{2H}(L)\pi_H\frac{\partial c_{2H}^{com}}{\partial \overline{c_{kH}}} - \frac{\gamma_{1H}}{\lambda}\alpha_{1H}(H)\pi_H\frac{\partial c_{1H}^{com}}{\partial \overline{c_{kH}}}$$
$$- \frac{\gamma_{2H}}{\lambda}\alpha_{2H}(H)\pi_H\frac{\partial c_{2H}^{com}}{\partial \overline{c_{kH}}} = \pi_H MWP_{kH} - \pi_H\sum_{j=1,2,3}\tau_j\frac{\partial c_{jH}^{com}}{\partial\overline{c_{kH}}}, \tag{A11}$$

$k = 1, 2$. Transforming A9 and A11 (4 equations) to matrix notation, one obtains equation (11).

Derivation of optimal commodity tax rates

Take the first-order condition for τ_j given by A5 and plug in for $\frac{\partial v_i}{\partial \tau_j} = -c_{ji}\frac{\partial v_i}{\partial x_i}$, $\frac{\partial v_H[L]}{\partial \tau_j} = -c_{jH}[L]\frac{\partial v_H[L]}{\partial x_L}$ and for the Slutsky-equations $\frac{\partial c_{ji}}{\partial \tau_j} = \frac{\partial c_{ji}^{com}}{\partial p_j} - c_{ji}\frac{\partial c_{ji}}{\partial x_i}$ and $\frac{\partial c_{mi}}{\partial \tau_j} = \frac{\partial c_{mi}^{com}}{\partial p_j} - c_{ji}\frac{\partial c_{mi}}{\partial x_i}$

with $j, m = 1, 2, 3$ and $j \neq m$. One gets

$$
\begin{aligned}
& - c_{jL} f_L \frac{\partial v_L}{\partial x_L} - c_{jH} f_H \frac{\partial v_H}{\partial x_H} + \lambda \sum_i \pi_i c_{ji} + \lambda \sum_{m=1,2,3} \sum_i \tau_m \pi_i (\frac{\partial c_{mi}^{com}}{\partial p_j} - c_{ji} \frac{\partial c_{mi}}{\partial x_i}) \\
& - \mu (\frac{\partial v_H}{\partial x_H} c_{jH} - \frac{\partial v_H[L]}{\partial x_L} c_{jH}[L]) - \gamma_{1L} \sum_i \alpha_{1i}(L) \pi_i (\frac{\partial c_{1i}^{com}}{\partial p_j} - c_{ji} \frac{\partial c_{1i}}{\partial x_i}) \\
& - \gamma_{1H} \sum_i \alpha_{1i}(H) \pi_i (\frac{\partial c_{1i}^{com}}{\partial p_j} - c_{ji} \frac{\partial c_{1i}}{\partial x_i}) - \gamma_{2L} \sum_i \alpha_{2i}(L) \pi_i (\frac{\partial c_{2i}^{com}}{\partial p_j} - c_{ji} \frac{\partial c_{2i}}{\partial x_i}) \\
& - \gamma_{2H} \sum_i \alpha_{2i}(H) \pi_i (\frac{\partial c_{2i}^{com}}{\partial p_j} - c_{ji} \frac{\partial c_{2i}}{\partial x_i}) = 0. \qquad \text{(A12)}
\end{aligned}
$$

Substituting for $f_L \frac{\partial v_L}{\partial x_L}$ and $f_H \frac{\partial v_H}{\partial x_H}$ from A1 and A2, equation A12 can be written as

$$
\begin{aligned}
& \sum_{m=1,2,3} \sum_i \tau_m \pi_i \frac{\partial c_{mi}^{com}}{\partial p_j} = - \frac{\mu}{\lambda} \frac{\partial v_H[L]}{\partial x_L} (c_{jH}[L] - c_{jL}) + \frac{\gamma_{1L}}{\lambda} \sum_i \alpha_{1i}(L) \pi_i \frac{\partial c_{1i}^{com}}{\partial p_j} \\
& + \frac{\gamma_{1H}}{\lambda} \sum_i \alpha_{1i}(H) \pi_i \frac{\partial c_{1i}^{com}}{\partial p_j} + \frac{\gamma_{2L}}{\lambda} \sum_i \alpha_{2i}(L) \pi_i \frac{\partial c_{2i}^{com}}{\partial p_j} + \frac{\gamma_{2H}}{\lambda} \sum_i \alpha_{2i}(H) \pi_i \frac{\partial c_{2i}^{com}}{\partial p_j}, \qquad \text{(A13)}
\end{aligned}
$$

$j = 1, 2, 3$. There are three equations with three unknowns τ_1, τ_2, τ_3. A13 can be transformed to matrix notation as given by equation (12). Applying Cramer's rule one arrives after some transformations at equations (14) and (17), where ψ_1, ψ_2, ψ_3 are given by

$$
\begin{aligned}
\psi_1 \equiv \; & \frac{\mu}{\lambda} \frac{\partial v_H[L]}{\partial x_L} [-(c_{1H}[L] - c_{1L}) (\sum_i \pi_i \frac{\partial c_{2i}^{com}}{\partial p_2} \sum_i \pi_i \frac{\partial c_{3i}^{com}}{\partial p_3} - \sum_i \pi_i \frac{\partial c_{2i}^{com}}{\partial p_3} \sum_i \pi_i \frac{\partial c_{3i}^{com}}{\partial p_2}) \\
& + (c_{2H}[L] - c_{2L}) (\sum_i \pi_i \frac{\partial c_{2i}^{com}}{\partial p_1} \sum_i \pi_i \frac{\partial c_{3i}^{com}}{\partial p_3} - \sum_i \pi_i \frac{\partial c_{3i}^{com}}{\partial p_1} \sum_i \pi_i \frac{\partial c_{2i}^{com}}{\partial p_3}) \\
& + (c_{3H}[L] - c_{3L}) (\sum_i \pi_i \frac{\partial c_{2i}^{com}}{\partial p_2} \sum_i \pi_i \frac{\partial c_{3i}^{com}}{\partial p_1} - \sum_i \pi_i \frac{\partial c_{2i}^{com}}{\partial p_1} \sum_i \pi_i \frac{\partial c_{3i}^{com}}{\partial p_2})], \qquad \text{(A14)}
\end{aligned}
$$

$$
\begin{aligned}
\psi_2 \equiv \; & \frac{\mu}{\lambda} \frac{\partial v_H[L]}{\partial x_L} [-(c_{2H}[L] - c_{2L}) (\sum_i \pi_i \frac{\partial c_{1i}^{com}}{\partial p_1} \sum_i \pi_i \frac{\partial c_{3i}^{com}}{\partial p_3} - \sum_i \pi_i \frac{\partial c_{1i}^{com}}{\partial p_3} \sum_i \pi_i \frac{\partial c_{3i}^{com}}{\partial p_1}) \\
& + (c_{1H}[L] - c_{1L}) (\sum_i \pi_i \frac{\partial c_{1i}^{com}}{\partial p_2} \sum_i \pi_i \frac{\partial c_{3i}^{com}}{\partial p_3} - \sum_i \pi_i \frac{\partial c_{3i}^{com}}{\partial p_2} \sum_i \pi_i \frac{\partial c_{1i}^{com}}{\partial p_3}) \\
& + (c_{3H}[L] - c_{3L}) (\sum_i \pi_i \frac{\partial c_{1i}^{com}}{\partial p_1} \sum_i \pi_i \frac{\partial c_{3i}^{com}}{\partial p_2} - \sum_i \pi_i \frac{\partial c_{3i}^{com}}{\partial p_1} \sum_i \pi_i \frac{\partial c_{1i}^{com}}{\partial p_2})], \qquad \text{(A15)}
\end{aligned}
$$

$$\psi_3 \equiv \frac{\mu}{\lambda}\frac{\partial v_H[L]}{\partial x_L}[-(c_{3H}[L]-c_{3L})(\sum_i \pi_i \frac{\partial c_{1i}^{com}}{\partial p_1}\sum_i \pi_i \frac{\partial c_{2i}^{com}}{\partial p_2} - \sum_i \pi_i \frac{\partial c_{1i}^{com}}{\partial p_2}\sum_i \pi_i \frac{\partial c_{2i}^{com}}{\partial p_1})$$
$$+(c_{1H}[L]-c_{1L})(\sum_i \pi_i \frac{\partial c_{2i}^{com}}{\partial p_2}\sum_i \pi_i \frac{\partial c_{1i}^{com}}{\partial p_3} - \sum_i \pi_i \frac{\partial c_{1i}^{com}}{\partial p_2}\sum_i \pi_i \frac{\partial c_{2i}^{com}}{\partial p_3})$$
$$+(c_{2H}[L]-c_{2L})(\sum_i \pi_i \frac{\partial c_{1i}^{com}}{\partial p_1}\sum_i \pi_i \frac{\partial c_{2i}^{com}}{\partial p_3} - \sum_i \pi_i \frac{\partial c_{2i}^{com}}{\partial p_1}\sum_i \pi_i \frac{\partial c_{1i}^{com}}{\partial p_3})]. \quad \text{(A16)}$$

Derivation of optimal marginal income tax rates

Divide the first order condition for z_H and x_H (A4 and A2) to get

$$\frac{(f_H+\mu)\frac{\partial v_H}{\partial z_H}}{(f_H+\mu)\frac{\partial v_H}{\partial x_H}} = \frac{-\lambda\pi_H - \lambda\pi_H\sum_j \tau_j \frac{\partial c_{jH}}{\partial z_H} + \pi_H \frac{\partial c_{1H}}{\partial z_H}\sum_i \gamma_{1i}\alpha_{1H}(i) + \pi_H\frac{\partial c_{2H}}{\partial z_H}\sum_i \gamma_{2i}\alpha_{2H}(i)}{\lambda\pi_H - \lambda\pi_H\sum_j \tau_j \frac{\partial c_{jH}}{\partial x_H} + \pi_H \frac{\partial c_{1H}}{\partial x_H}\sum_i \gamma_{1i}\alpha_{1H}(i) + \pi_H\frac{\partial c_{2H}}{\partial x_H}\sum_i \gamma_{2i}\alpha_{2H}(i)}. \quad \text{(A17)}$$

Use the definition $MRS^i_{zx} \equiv -\frac{\partial v_i/\partial z_i}{\partial v_i/\partial x_i}$ and rewrite A17 as

$$-MRS^H_{zx}(\lambda\pi_H - \lambda\pi_H\sum_j \tau_j \frac{\partial c_{jH}}{\partial x_H} + \pi_H\frac{\partial c_{1H}}{\partial x_H}\sum_i \gamma_{1i}\alpha_{1H}(i) + \pi_H\frac{\partial c_{2H}}{\partial x_H}\sum_i \gamma_{2i}\alpha_{2H}(i)) =$$
$$-\lambda\pi_H - \lambda\pi_H\sum_j \tau_j\frac{\partial c_{jH}}{\partial z_H} + \pi_H\frac{\partial c_{1H}}{\partial z_H}\sum_i \gamma_{1i}\alpha_{1H}(i) + \pi_H\frac{\partial c_{2H}}{\partial z_H}\sum_i \gamma_{2i}\alpha_{2H}(i) \quad \text{(A18)}$$

After dividing by $\lambda\pi_H$ A18 can be rewritten as

$$1 - MRS^H_{zx} = \sum_{k=1,2}(\frac{\gamma_{kL}}{\lambda}\alpha_{kH}(L) + \frac{\gamma_{kH}}{\lambda}\alpha_{kH}(H) - \tau_k)(\frac{\partial c_{kH}}{\partial z_H} + MRS^H_{zx}\frac{\partial c_{kH}}{\partial x_H})$$
$$-\tau_3(\frac{\partial c_{3H}}{\partial z_H} + MRS^H_{zx}\frac{\partial c_{3H}}{\partial x_H}) \quad \text{(A19)}$$

Useing equation (5) yields equation (21) from the text. To arrive at the optimal marginal income tax for the L types, first divide A3 by A1 to get

$$\frac{f_L\frac{\partial v_L}{\partial z_L}}{f_L\frac{\partial v_L}{\partial x_L}} = \frac{-\lambda\pi_L - \lambda\pi_L\sum_j\tau_j\frac{\partial c_{jL}}{\partial z_L} + \mu\frac{\partial v_H[L]}{\partial z_L} + \pi_L\frac{\partial c_{1L}}{\partial z_L}\sum_i \gamma_{1i}\alpha_{1L}(i) + \pi_L\frac{\partial c_{2L}}{\partial z_L}\sum_i\gamma_{2i}\alpha_{2L}(i)}{\lambda\pi_L - \lambda\pi_L\sum_j\tau_j\frac{\partial c_{jL}}{\partial x_L} + \mu\frac{\partial v_H[L]}{\partial x_L} + \pi_L\frac{\partial c_{1L}}{\partial x_L}\sum_i \gamma_{1i}\alpha_{1L}(i) + \pi_L\frac{\partial c_{2L}}{\partial x_L}\sum_i\gamma_{2i}\alpha_{2L}(i)}. \quad \text{(A20)}$$

Again use the definition for the MRS^L_{zx} from above and divide A20 by $\lambda\pi_L$ and $\partial v_H[L]/\partial x_L$. Then

A20 can be rewritten as

$$-\frac{MRS_{zx}^{L}}{\partial v_H[L]/\partial x_L}(1-\sum_j \tau_j \frac{\partial c_{jL}}{\partial x_L}+\frac{\mu}{\lambda \pi_L}\frac{\partial v_H[L]}{\partial x_L}+\frac{\partial c_{1L}}{\partial x_L}\sum_i \frac{\gamma_{1i}}{\lambda}\alpha_{1L}(i)+\frac{\partial c_{2L}}{\partial x_L}\sum_i \frac{\gamma_{2i}}{\lambda}\alpha_{2L}(i))=$$
$$\frac{1}{\partial v_H[L]/\partial x_L}(-1-\sum_j \tau_j \frac{\partial c_{jL}}{\partial z_L}+\frac{\partial c_{1L}}{\partial z_L}\sum_i \frac{\gamma_{1i}}{\lambda}\alpha_{1L}(i)+\frac{\partial c_{2L}}{\partial z_L}\sum_i \frac{\gamma_{2i}}{\lambda}\alpha_{2L}(i))-\frac{\mu}{\lambda \pi_L}MRS_{zx}^{H}[L] \tag{A21}$$

Multiply A21 by $\partial v_H[L]/\partial x_L$ and use equation (5) to get equation (22) from the text.

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