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Bayesian Treatment Effects Models with Variable Selection for Panel Outcomes with an Application to Earnings Effects of Maternity Leave[1]

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Abstract

Child birth leads to a break in a woman's employment history and is considered one reason for the relatively poor labor market outcomes observed for women compared to men. However, the time spent at home after child birth varies significantly across mothers and is likely driven by observed and, more importantly, unobserved factors that also affect labor market outcomes directly. In this paper we propose

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two alternative Bayesian treatment modeling and inferential frameworks for panel outcomes to estimate dynamic earnings effects of a long maternity leave on mothers' earnings in the years following the return to the labor market. The frameworks differ in their modeling of the endogeneity of the treatment and the panel structure of the earnings, with the first framework based on the modeling tradition of the Roy switching regression model, and the second based on the shared factor approach. We show how stochastic variable selection can be implemented within both frameworks and can be used, for example, to test for the heterogeneity of the treatment effects. Our analysis is based on a large sample of mothers from the Austrian Social Security Register (ASSD) and exploits a recent change in the maternity leave policy to help identify the causal earnings effects. We find substantial negative earnings effects from long leave over a 5 years period after mothers' return to the labor market, with the earnings gap between short and long leave mothers steadily narrowing over time.

1 Introduction

A robust finding that has emerged from a vast literature is that mothers earn less than women without children. According to empirical work a significant part of this motherhood wage penalty, a key reason for the observed gender wage gap, can be explained by lower human capital, in particular years out of the labor force and unobserved heterogeneity (Anderson, Binder, and Krause, 2002; Budig and England, 2001; Waldfogel, 1998a,b; Lundberg and Rose, 2000). The lower human capital of mothers partly results from a break in the employment history as women stay at home for some period of time after child birth to care for the newborn (maternity leave). This break is likely to lead to a depreciation of general and firm-specific skills during absences from the labor market and lost rents associated with good job matches.

However, the amount of time mothers spend at home before returning to the labor market varies considerably, even among those covered by the same maternity leave policy. A mother's decision when to return to the labor market depends on a range of additional factors and is likely driven by observed and more importantly unobserved factors that also affect labor market outcomes directly. In this paper we investigate the effect of a long maternity leave on a mother's earnings after her return to the labor market. We introduce two Bayesian treatment effects modeling frameworks for panel outcomes to estimate the causal earnings effects of the endogenous leave treatment from a large sample of mothers created from a unique Austrian registry data set.

The estimation of treatment effects has become a focus of many econometric papers, in particular the identification and estimation of the effect of an endogenous treatment variable on some outcome of interest. Several approaches have been popular to identify causal treatment effects in such settings, in particular instrumental variable approaches, the LATE estimator and joint modeling approaches (Lee, 2005; Heckman, Ichimura, and Todd, 1998; Heckman and Navarro-Lozano, 2004). Bayesian inferential methods to treatment effect estimation are commonly based on some flexible joint modeling approach, often in the spirit of Roy's switching regression model (Roy, 1951; Lee, 1978) and have addressed a range of issues such as panel outcomes and heterogeneity in treatment across subjects (Koop and Poirier, 1997; Chib and Hamilton, 2000; Munkin and Trivedi, 2003; Chib, 2007; Chib and Jacobi, 2007; Li and Tobias, 2011).

Building on this literature, we introduce two modeling frameworks within the Bayesian paradigm to estimate the causal effect of an endogeneous binary treatment on panel outcomes. Both models are formulated within the potential outcome framework following the standard approach in the treatment literature. The first framework is formulated in the tradition of Bayesian treatment effects models in terms of a joint modeling framework for the treatment and the potential outcomes based on the Roy switching regression model (Roy, 1951; Lee, 1978) to capture the endogeneity of the treatment, and does not require the specification of the unidentified joint distribution of the two potential outcome sequences. We discuss two alternatives to model the dependence across the panel outcomes. The second framework employs the more recent factor approach to model the endogeneity of the treatment as well as the panel structure of the earnings following Carneiro, Hansen, and Heckman (2003). Both frameworks contain flexible formulations of the potential outcomes to capture heterogeneous treatment effects, allowing for different effects of the treatment across subjects and different time dynamics in the two treatment groups.

As an additional innovative and useful feature of these frameworks we introduce Bayesian variable selection in the context of treatment effects models, which has been implemented in many Bayesian papers in the context of "non-treatment" models (for example George and McCulloch (1993, 1997); Geweke (1996); Ley and Steel (2009); Frühwirth-Schnatter and Wagner (2010)). This feature together with a suitable specification of the model will enable us to determine which covariates should be included in the model and to test for the existence of common and level-specific effects of the treatment as well as covariates.

In our analysis we also exploit a recent exogenous change in the parental leave policy in Austria to help identify the causal labor earnings effects of the endogenously determined leave time. In July 2000 Austria extended the benefit period from 18 months since the birth of the child to 30 months. The period of job protection remained unchanged at 24 months. This exogenous policy change lead to an exogenous variation in time mothers spent at home (Lalive, Schlosser, Steinhauer, and Zweimüller, 2014). Previous analysis of mothers in the US and Britain has indicated that having access to job protected maternity leave has a positive wage effect for mothers by increasing the likelihood that the mother returns to the same employer thus decreasing the loss of firm-specific skills and the maintaining of good job matches. It has also been argued (Waldfogel, 1998a) that job-protected maternity leave may have a negative wage effect by inducing women to take a longer leave than otherwise, leading to a loss in job experience. A recent study of a change in the Austrian parental leave policy (Lalive and Zweimüller, 2009), finds that the extension of the job and benefit period from 12 months to 24 months delays return to work and has negative short-term consequences for wages.

The data for our analysis comes from the Austrian Social Security Register (ASSD), an administrative individual register data that collects information for old-age security benefits (Zweimüller, Winter-Ebmer, Lalive, Kuhn, Wuellrich, Ruf, and Büchi, 2009). The data set has several advantages. In addition to the global coverage, we have precise information whether and how long a mother took maternity leave and whether she returned to the same employer. A weakness of the data is the lack of information on hours worked which we have taken into account in the modeling of the data, for example by allowing for flexible time dynamics and dependence structure in the panel outcomes.

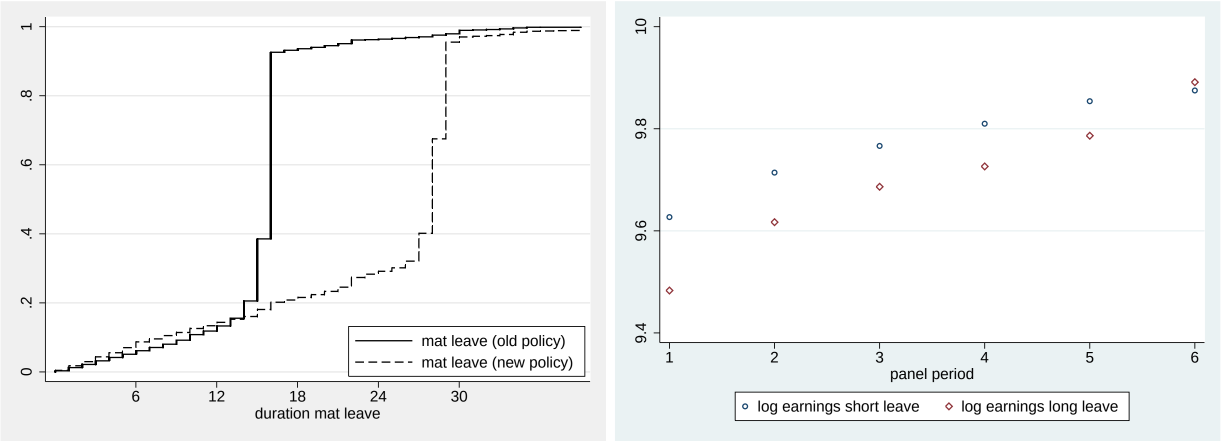
The remainder of the paper is organized as follows. In Section 2 we provide some background about the maternity policy change in Austria. Section 3 describes our modeling frameworks and in Section 4 we discuss Bayesian inference including variable selection. Section

5 contains our simulation study and Section 6 the empirical analysis. We finish with the conclusions in Section 7.

2 Background: Parental Leave Policy Maternity Leave Austria

In Austria, the first mothers return to work 2 months after the birth of the child which is the end of the standard mother protection period. The parental leave policy starts after the end of this period. In Austria, the parental leave policy has two components: job protection and the payment of parental leave benefits. Since July 1990, the job protection and leave benefits periods were extended from previously 12 months since the birth of the child to 24 months. The length of the benefits payment period has undergone several changes more recently. A reduction to 18 months in July 1996 has been followed by an extension of the leave period to up to 30 months, 6 months beyond the job protection period, in July 2000.

The extension of the benefits period by one year and beyond the job protection period in July 2000 induced a substantial proportion of mothers to delay return to work. Panel (a) in Figure 1 shows the empirical cdfs of the duration of leave after child birth by policy regime based on a sample of mothers who gave birth in a 2 year window before and after the 2000 policy change. The graph is based on a sample of mothers taken from the Austrian Social Security Register (ASSD) which contains the complete individual employment histories for the universe of Austrian employees since 1972, including information on number of births and maternity and parental leave spells. The mothers could not predict the policy change as it was made public on August 7, 2001 with an effective date of January 1st 2002. Further, to ensure equal treatment of mothers who were on leave August 7, 2001 and gave birth after July 1st, 2000 they could extend the job protected leave to 2 years and parental leave payments to 30 months. As we can see from panel (a), mothers start to return to work after the end of the mother protection period, within each group the majority of mothers return in the months leading up to the end of the benefit period under the relevant policy scheme. Under the old



(a) Cdf of Maternity Leave by Policy

(b) Average Log Earnings by Leave

Figure 1: Empirical cdf of the duration of leave after the child birth and average log earnings for mothers with short and long leave by panel period

policy regime a large proportion of mothers return just before month 18, while under the new policy regime most mothers return just before month 30.

We therefore consider two groups of mothers based on their leave, those with a maternity leave up to 18 months (short leave) and those with a maternity leave beyond 18 month (long leave). This paper focuses on the identification of the effect of a long versus a short maternity leave, the binary treatment, on the subsequent earnings of mothers following their return to the labor market. As discussed, there are several potential reasons to believe that mothers with a longer maternity leave receive lower earnings at their return to the labor market such as a higher loss of human capital and loss of good job matches. Panel (b) in Figure 1 shows the average log yearly earnings for mothers in both leave groups for six consecutive panel periods (years) following their return to the labor market. The graph suggests that mothers with longer leave start out with substantially lower earnings in their first full year in the labor market than mothers with a short leave, and continue to earn less in the 5 years after their return before the gap closes.

However, we have to be careful with the interpretation of the differences in terms of earnings effects as we do not account for the endogenous choice of the maternity leave state that may affect the earnings across the two groups. While a mother's choice of length of maternity leave is heavily affected by the length of the benefit period under the policy regime in place, their choice also depends on a range of unobserved factors related to later yearly earnings such as availability and attitudes to child care and personal investment in child rearing after their return to the labor market. The latter might have strong affects on hours worked after a mother's return and thus yearly earnings. Further, while the graph compares mothers across the two treatment groups in the same panel period, the data points across the two treatment groups also contain calender year effects.

3 Alternative Treatment Model Specifications for Panel Outcomes

In this section we introduce two alternative Bayesian treatment modeling frameworks to isolate the effect of long maternity leave on panel earnings. Both are phrased within the potential outcome framework commonly used in the treatment literature, allowing for a heterogeneous panel treatment effects on earnings, and specify joint regression type models for selection into treatment and the panel outcomes. The differences in the modeling frameworks are with respect to modeling second order moments such as variance and dependence needed to capture the key features of the data. While the first framework is based on the switching regression approach used previously, the second framework introduces a factor model approach to modeling of treatment effect data.

In the following we first formally describe the data structure motivated by our application in terms of useful notation, before introducing the mean structure for the selection into treatment and the potential outcome sequences common across the two modeling frameworks. We then specify switching regression and shared factor models as alternative approaches to capture the dependence across the outcomes and the dependence between the treatment and discuss their implications for the observed data models and treatment effects.

3.1 Data Structure and Notation

Consider the following setting based on the empirical example with a sample of $i = 1, ..., n$ mothers that gave birth between July 1998 and June 2002. Following from the previous discussion we define the exogenous binary policy variable z_i, our instrumental variable, for each mother as

$$z_i = \begin{cases} 0, & \text{if child born before July 2000,} \\ 1, & \text{if child born after June 2000.} \end{cases}$$

As discussed before, mothers with $z_i = 0$ receive maternity leave benefits up to 18 months, while mothers with $z_i = 1$ receive benefits up to 30 months. Job protection ends at 24 months under both policy regimes.

We let the variable m_i denote the number of months a mother spends on maternity leave before returning to the labor market and define the endogeneous binary maternity leave treatment variable x_i for short (0) or long (1) leave as

$$x_i = \begin{cases} 0, & \text{if } m_i \leq 18, \\ 1, & \text{if } m_i > 18. \end{cases}$$

Each mother has a vector $\mathbf{v}_i$ of baseline characteristics at treatment, such as demographic characteristics and earnings before maternity leave, that affect selection into the treatment in addition to the policy regime in place at the time.

For each mother we further observe a vector of labor market outcomes $\mathbf{y}_i = \{y_{i1}, y_{i2}, ..., y_{iT_i}\}$, here measured as log earnings. The time subscript refers to the period (year) since return to the labor market where T_i denotes the number of consecutive panel periods (years) for which we observe the mother i in the labor market after her return.

We also observe a matrix of demographic and job related variables that affect earnings, $\mathbf{W}_i = \{\mathbf{w}_{i1}, \mathbf{w}_{i2}, ..., \mathbf{w}_{iT_i}\}$ with some elements varying over time. For the sake of a simpler notation and clearer description of the model specifications and features we assume a balanced panel with T yearly observations for each mother in following discussion.

For each mother we define the two potential outcome sequences of log earnings under the two possible treatments as $\mathbf{y}_{0i}$ and $\mathbf{y}_{1i}$, with $\mathbf{y}_{0i} = \{y_{0,i1}, y_{0,i2}, ..., y_{0,iT}\}$ referring to the vector of potential earnings under short leave ($x_i = 0$) and $\mathbf{y}_{1i} = \{y_{1,i1}, y_{1,i2}, ..., y_{1,iT}\}$ to potential earnings vector under long leave ($x_i = 1$). Depending on the realized treatment x_i, we observe only one of these potential outcome sequences, i.e.:

$$\mathbf{y}_i = \mathbf{y}_{0i}(1 - x_i) + \mathbf{y}_{1i} x_i.$$

Due to this data restriction, the correlation between the potential outcomes and thus individual level treatment effects $(\mathbf{y}_{1i} - \mathbf{y}_{0i} | \mathbf{W}_i)$ cannot be identified from the data without additional identification assumptions that can never be verified by the data. Hence the treatment effect literature focuses on the estimation of the average causal treatment effect (ATE)

$$\text{ATE}(\mathbf{W}) = E[\mathbf{y}_{1i} | \mathbf{W}] - E[\mathbf{y}_{0i} | \mathbf{W}],$$

for a particular matrix $\mathbf{W}$ of demographic and job related variables. Further treatment effects such as the average causal treatment effect on the treated and untreated can be identified and estimated only under additional unverifiable assumptions on the joint distribution of the potential outcome sequences.

3.2 Modeling the Mean Structures of the Endogenous Leave Treatment and Potential Earnings Sequences

As noted above, the two modelling approaches differ only with respect to modelling second order moments such as dependencies and variances structures. In both approaches, selection into the endogenous treatment x_i is specified as a standard probit model via a normal latent variable as $x_i = I\{x_i^* > 0\}$ with

$$x_i^* = \mathbf{v}_i'\boldsymbol{\alpha}_1 + z_i\alpha_2 + \eta_i \ , \qquad \eta_i \sim \mathcal{N}\left(0, \sigma_x^2\right), \tag{3.1}$$

where z_i is the instrument based on the policy regime defined above. We will denote by $\mu(x_i^*) = \mathbf{v}_i'\boldsymbol{\alpha}_1 + z_i\alpha_2$ the conditional expectation of x_i^* given the covariates $(\mathbf{v}_i, z_i)$. Subsequently, we will refer to x_i^* as latent utility.

To capture the heterogeneous effect of long maternity leave on log earnings we specify the basic observation models for the two potential outcome vectors $\mathbf{y}_{0i}$ and $\mathbf{y}_{1i}$ as

$$\mathbf{y}_{0i} = \mathbf{1}_T\mu + \mathbf{W}_i\boldsymbol{\gamma} + \boldsymbol{\varepsilon}_{0i}, \tag{3.2}$$

$$\mathbf{y}_{1i} = \mathbf{1}_T(\mu + \kappa) + \mathbf{W}_i(\boldsymbol{\gamma} + \boldsymbol{\theta}) + \boldsymbol{\varepsilon}_{1i}, \tag{3.3}$$

where $\mathbf{y}_{0i}$ and $\mathbf{y}_{1i}$ refers to the potential log earnings of a mother under a short leave and long leave, respectively. This general formulation assumes that all covariate effects can vary with the treatment. We have a common and heterogeneous effects of the treatment captured by the coefficients κ and $\boldsymbol{\theta}$, respectively. This specification is also useful when we introduce variable selection into the modeling framework to test for the presence of the common and heterogenous treatment effects. To simplify notation, we will use $\mu(\mathbf{y}_{ji})$ to denote the conditional expectation of $\mathbf{y}_{ji}$ for $j = 0, 1$ given the covariates $\mathbf{W}_i$, i.e. $\mu(\mathbf{y}_{0i}) = \mathbf{1}_T\mu + \mathbf{W}_i\boldsymbol{\gamma}$ and $\mu(\mathbf{y}_{1i}) = \mathbf{1}_T(\mu+\kappa) + \mathbf{W}_i(\boldsymbol{\gamma} + \boldsymbol{\theta})$.

3.3 Modeling the Dependence and Variance Structures

Further assumptions concerning the unobserved errors η_i in the selection equation (3.1) and the composite errors $\boldsymbol{\varepsilon}_{ji}$ in the two potential outcome equations (3.2) and (3.3) are necessary for identification. First of all, we assume that the composite errors $\boldsymbol{\varepsilon}_{ji}$ follow a normal distribution, $\boldsymbol{\varepsilon}_{ji} \sim \mathcal{N}_T\left(0, \Omega_j\right)$, where the covariance matrices Ω_j capture dependence between subsequent outcomes under treatment j.

A key feature and modeling challenge is the dependence between the treatment and the outcomes. Mothers choose the length of the maternity leave based on a range of considerations, including their job market prospects and unobserved factors related to subsequent earnings, implying that treatment is endogenous. This endogeneity can be captured by specifying a joint distribution for the error terms in the treatment and potential outcome equations $(\boldsymbol{\varepsilon}_{0i}, \boldsymbol{\varepsilon}_{1i}, \eta_i)$ as

$$\begin{pmatrix} \boldsymbol{\varepsilon}_{0i} \\ \boldsymbol{\varepsilon}_{1i} \\ \eta_i \end{pmatrix} \sim \mathcal{N}_{2T+1}\left(\mathbf{0}, \begin{pmatrix} \Omega_0 & \Omega_{01} & \boldsymbol{\omega}_0 \\ \Omega_{01} & \Omega_1 & \boldsymbol{\omega}_1 \\ \boldsymbol{\omega}_0' & \boldsymbol{\omega}_1' & \sigma_x^2 \end{pmatrix}\right).$$

As we never observe both sequences of potential outcomes for one individual, the covariance matrix Ω_{01} of the potential outcome sequences cannot be identified directly from the observed data. While it is possible to identify bounds on the covariance of the potential outcomes based on the positive-definiteness restriction and the remaining components of the covariance matrix of the three error terms in the simple cross-section case as shown in Koop and Poirier (1997), such an undertaking appears to be infeasible for our larger (unbalanced) panel data case.

We will consider two modelling approaches, which differ in the concrete specification of the structure of the joint covariance matrix $\mathsf{Cov}(\boldsymbol{\varepsilon}_{0i}, \boldsymbol{\varepsilon}_{1i}, \eta_i)$. As shown in Chib (2007), identification of Ω_{01} can be avoided by specifying only the $(T+1)$-variate distribution $(\boldsymbol{\varepsilon}_{ji}, \eta_i)$ for each subject for both treatments. This is our first modelling approach which is based on the switching regression model, also referred to as the Roy model. In the second model, the shared factor model, we will assume that all three errors $(\boldsymbol{\varepsilon}_{0i}, \boldsymbol{\varepsilon}_{1i}, \eta_i)$ share a common latent factor to which all correlation between these terms can be attributed.

3.3.1 Switching Regression Models

In the switching regression model (SR) we have to specify only the structure of $\boldsymbol{\omega}_j$ and Ω_j for both treatments. To do so, we consider two variants of the SR model. In a first variant, the SRI model, dependence between subsequent outcomes is captured by introducing an individual- and treatment-specific random intercept $b_{ji} \sim \mathcal{N}(0, D_j)$ and specifying, both for $j = 0, 1$,

$$\boldsymbol{\varepsilon}_{ji} = \mathbf{1}_T b_{ji} + \boldsymbol{\epsilon}_{ji}, \tag{3.4}$$

where b_{ji} is assumed to be independent of the errors $\boldsymbol{\epsilon}_{ji}$, $\boldsymbol{\epsilon}_{ji} \sim \mathcal{N}_T(0, \Sigma_j)$ and Σ_j is a diagonal matrix with elements $\boldsymbol{\sigma}_j^2 = (\sigma_{j,1}^2, \dots, \sigma_{j,T}^2)$. Marginalizing over the random intercept b_{ji}, the potential earnings vectors $\mathbf{y}_{ji}$ have a multivariate normal distribution with covariance matrix Ω_j defined as

$$\mathsf{Cov}(\mathbf{y}_{ji}) = \Omega_j = \Sigma_j + D_j \mathbf{1}_T \mathbf{1}_T' = \begin{pmatrix} \sigma_{j,1}^2 + D_j & D_j & \dots & D_j \\ D_j & \sigma_{j,2}^2 + D_j & \dots & D_j \\ \vdots & \vdots & \ddots & \vdots \\ D_j & D_j & \dots & \sigma_{j,T}^2 \end{pmatrix}.$$

The compound symmetry structure of the covariance matrix implies that the covariance between outcomes at different points in time remains constant.

As this assumption is potentially too restrictive for the data at hand, we consider as a more flexible alternative a latent factor structure of Ω_j. To specify the switching regression model with a latent factor (SRF) we introduce individual- and treatment-specific latent factors $\tilde{b}_{ji} \sim N(0, 1)$ and vectors of time-varying factor loadings $\boldsymbol{\lambda}_j$, to explain the covariance structure in $\boldsymbol{\varepsilon}_{ji}$:

$$\boldsymbol{\varepsilon}_{ji} = \boldsymbol{\lambda}_j \tilde{b}_{ji} + \boldsymbol{\epsilon}_{ji}, \tag{3.5}$$

where $\tilde{b}_{ji}$ is assumed to be independent of the idiosyncratic errors $\boldsymbol{\epsilon}_{ji}$. As above, the errors $\boldsymbol{\epsilon}_{ji}$ follow a normal distribution, $\boldsymbol{\epsilon}_{ji} \sim \mathcal{N}_T(0, \Sigma_j)$, with Σ_j being a diagonal matrix with elements $\boldsymbol{\sigma}_j^2 = (\sigma_{j,1}^2, \dots, \sigma_{j,T}^2)$. Marginalizing over the latent factors yields the following covariance matrix of the potential outcomes,

$$\mathsf{Cov}(\mathbf{y}_{ji}) = \Omega_j = \Sigma_j + \boldsymbol{\lambda}_j \boldsymbol{\lambda}_j' = \begin{pmatrix} \sigma_{j,1}^2 + \lambda_{j,1}^2 & \lambda_{j,1}\lambda_{j,2} & \dots & \lambda_{j,1}\lambda_{j,T} \\ \lambda_{j,1}\lambda_{j,2} & \sigma_{j,2}^2 + \lambda_{j,2}^2 & \dots & \lambda_{j,2}\lambda_{j,T} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{j,1}\lambda_{j,T} & \lambda_{j,2}\lambda_{j,T} & \dots & \sigma_{j,T}^2 + \lambda_{j,T}^2 \end{pmatrix}, \tag{3.6}$$

which obviously has a more flexible structure than in the SRI model. The compound symmetry structure of the SRI model arises as that special case where the factor loadings are constant, i.e. $\boldsymbol{\lambda}_j = \sqrt{D_j}\mathbf{1}_T$.

As shown by Chib (2007) it is sufficient to specify only the joint distribution of the errors in the selection model and the outcome sequence under treatment j if interest lies in the identification of the average treatment effect. This means that only the two $(T+1)$-variate distributions of the errors $(\boldsymbol{\varepsilon}_{ji}, \eta_i)$ have to be specified for both treatments $j = 0$ and $j = 1$:

$$\begin{pmatrix} \boldsymbol{\varepsilon}_{ji} \\ \eta_i \end{pmatrix} \sim \mathcal{N}_{T+1}\left(\mathbf{0}, \begin{pmatrix} \Omega_j & \boldsymbol{\omega}_j \\ \boldsymbol{\omega}_j' & 1 \end{pmatrix}\right), \quad j = 0, 1. \tag{3.7}$$

The restriction $\mathsf{V}(\eta_i) = \sigma_x^2 = 1$ is based on the standard identification argument for probit models.

We follow here Chib and Jacobi (2008) and model the dependence structure in (3.7) indirectly, by specifying both for the SRI model (3.4) as well as the SRF model (3.5), the $(T+1)$-variate joint distribution of η_i and the idiosyncratic errors $\boldsymbol{\epsilon}_{ji}$ in both outcome equations for as:

$$\begin{pmatrix} \boldsymbol{\epsilon}_{ji} \\ \eta_i \end{pmatrix} \sim \mathcal{N}_{T+1}\left(\mathbf{0}, \begin{pmatrix} \Sigma_j & \boldsymbol{\omega}_j \\ \boldsymbol{\omega}_j' & 1 \end{pmatrix}\right). \tag{3.8}$$

This covariance structure implies that $\boldsymbol{\omega}_j = \Sigma_j^{1/2}\boldsymbol{\rho}_j$, with correlation $\boldsymbol{\rho}_j = \mathsf{Cor}(\boldsymbol{\epsilon}_{ji}, \eta_i)$. No further structure is assumed for the correlations $\boldsymbol{\rho}_j = (\rho_{j,1}, \ldots, \rho_{j,T})$, however restrictions can arise from the assumption that $(\boldsymbol{\epsilon}_{ji}, \eta_i)$ has a proper $(T+1)$-variate normal distribution and hence the covariance matrix has to be positive definite.

We would like to emphasize here some specifics of this model: First, as indicated by the subscript j, the latent variables b_{ji} or $\tilde{b}_{ji}$ are specific to each potential outcome vector. Second, the error $\boldsymbol{\varepsilon}_{ji}$ of the potential outcome model is decomposed into the contribution of the latent variables, which captures only dependence within a vector of potential outcomes and a pure error which captures dependence between potential outcome and latent utility.

In the following we will denote the models as SRI and SRF if we want to emphasize the different covariance structures and as SR otherwise. Further, for simplicity we will use $\mathbf{b}$ to address the collection of random intercepts or latent factors for the observed outcomes, i.e. $\mathbf{b} = (b_{x_1,1}, \ldots, b_{x_n,n})$ in the SRI and $\mathbf{b} = (\tilde{b}_{x_1,1}, \ldots, \tilde{b}_{x_n,n})$ in the SRF.

3.3.2 Shared Factor Model

An alternative approach to model the panel dependence between outcomes and the treatment is the factor approach (Carneiro et al., 2003; Heckman, Lopes, and Piatek, 2014). We employ this approach to specify a shared factor model (SF) for the panel outcomes and the treatment selection by introducing a latent factor $f_i \sim N(0,1)$ that captures both the panel correlation within the potential outcome sequences, as well as the dependence between the outcomes and the treatment.

Keeping the mean structure of the treatment and potential outcome models as specified in Section 3.2 we now define the errors in all three models to include the shared latent factor f_i,

$$\eta_i = \lambda_x f_i + \upsilon_i, \quad \upsilon_i \sim \mathcal{N}(0,1), \tag{3.9}$$

$$\boldsymbol{\varepsilon}_{0i} = \boldsymbol{\lambda}_0 f_i + \boldsymbol{\epsilon}_{0i}, \quad \boldsymbol{\epsilon}_{0i} \sim \mathcal{N}_T(0, \Sigma_0), \tag{3.10}$$

$$\boldsymbol{\varepsilon}_{1i} = \boldsymbol{\lambda}_1 f_i + \boldsymbol{\epsilon}_{1i}, \quad \boldsymbol{\epsilon}_{1i} \sim \mathcal{N}_T\left(0, \boldsymbol{\Sigma}_1\right). \tag{3.11}$$

To identify all parameters in the model, we have fixed the variance of the random factor and the variance of the pure error in the selection model at 1, but do not restrict any of the factor loadings.

As in Section 3.2 we assume that the idiosyncratic errors $\boldsymbol{\epsilon}_{ji}$ follow a normal distribution, $\boldsymbol{\epsilon}_{ji} \sim \mathcal{N}_T\left(0, \boldsymbol{\Sigma}_j\right)$, with $\boldsymbol{\Sigma}_j$ being a diagonal matrix with elements $\boldsymbol{\sigma}_j^2 = (\sigma_{j,1}^2, \dots, \sigma_{j,T}^2)$, and that the factor f_i is independent of the idiosyncratic errors $\boldsymbol{\epsilon}_{ji}$ as well as υ_i. An implication of this specification is that $\mathsf{V}(\eta_i) = \sigma_x^2 = 1 + \lambda_x^2$. The covariance across the outcomes within a potential earnings sequence is given as $\mathsf{Cov}(\mathbf{y}_{ji}) = \Omega_j = \boldsymbol{\Sigma}_j + \boldsymbol{\lambda}_j \boldsymbol{\lambda}_j'$ which is identical to that of the SRF-model given in equation (3.6).

A particular implication of the shared factor structure in the treatment and outcome models is the implied dependence between potential outcome sequences. Marginalizing over the latent factor f_i the joint distribution of the error terms is given by the $(2T+1)$-variate normal distribution

$$\begin{pmatrix} \boldsymbol{\varepsilon}_{0i} \\ \boldsymbol{\varepsilon}_{1i} \\ \eta_i \end{pmatrix} \sim \mathcal{N}_{2T+1}\left(\mathbf{0}, \begin{pmatrix} \boldsymbol{\Sigma}_0 + \boldsymbol{\lambda}_0\boldsymbol{\lambda}_0' & \boldsymbol{\lambda}_0\boldsymbol{\lambda}_1' & \lambda_x\boldsymbol{\lambda}_0 \\ \boldsymbol{\lambda}_1\boldsymbol{\lambda}_0' & \boldsymbol{\Sigma}_1 + \boldsymbol{\lambda}_1\boldsymbol{\lambda}_1' & \lambda_x\boldsymbol{\lambda}_1 \\ \lambda_x\boldsymbol{\lambda}_0' & \lambda_x\boldsymbol{\lambda}_1' & 1 + \lambda_x^2 \end{pmatrix}\right), \tag{3.12}$$

which implies that the covariance between the potential outcome sequences is given by $\mathsf{Cov}(\mathbf{y}_{0i}, \mathbf{y}_{1i}) = \boldsymbol{\lambda}_0\boldsymbol{\lambda}_1'$. Further, the shared factor model implies that the dependence between the treatment and the outcomes resulting from the endogeneity of the treatment is captured by $\mathsf{Cov}(\mathbf{y}_{ji}, x_i^*) = \boldsymbol{\lambda}_j\lambda_x$. Note that, though the signs of factor f_i and factor loadings $\lambda_x, \boldsymbol{\lambda}_j$ are not fully identified since the likelihood for $\lambda_x f_i$ is the same as the likelihood for $(-\lambda_x)(-f_i)$, and also the likelihoods for $\boldsymbol{\lambda}_j f_i$ and $(-\boldsymbol{\lambda}_j)(-f_i)$ are equal, the signs of all elements in the joint covariance matrix are identified. The correlation between x_i^* and $y_{j,it}$ is given by

$$\mathsf{Cor}(x_i^*, y_{j,it}) = \frac{\lambda_{j,t}\lambda_x}{\sqrt{1+\lambda_x^2}\sqrt{\sigma_{j,t}^2 + \lambda_{j,t}^2}}. \tag{3.13}$$

In comparison, the SR model specified in the previous section allows for a more flexible structure in the dependence between the treatment and the outcomes and as a result may provide a better fit for the data at hand. Also, since we never observe both outcome sequences for one individual, an advantage of the SR model is that it does not make any assumptions about the joint distributions of the potential outcome sequences that cannot be verified by the data. However, an advantage of modeling the dependence between the potential outcome sequences in the shared factor model is that it enables us to estimate treatment effects beyond the average treatment effect, for example the treatment effects on the treated and untreated, see Heckman et al. (2014).

3.4 Observed data models

A specific feature of treatment effects models is their specification in terms of unobserved variables: The potential outcome sequence $\mathbf{y}_{0i}$ is observed only for $x_i = 0$ corresponding to $x_i^* < 0$, whereas for $x_i = 1$ corresponding to $x_i^* > 0$ only $\mathbf{y}_{1i}$ is observed. Moreover the latent utility is never observed. We investigate now the implications of the models for the observed data.

In both modelling approaches the joint distributions of the potential earnings and the latent utility, $p(\mathbf{y}_{0i}, x_i^*)$ and $p(\mathbf{y}_{1i}, x_i^*)$ are $(T+1)$-variate normal distributions, given as

$$(\mathbf{y}_{ji}, x_i^*) \sim \mathcal{N}_{T+1}\left(\begin{pmatrix}\mu(\mathbf{y}_{ji})\\ \mu(x_i^*)\end{pmatrix}, \begin{pmatrix}\Omega_j & \Sigma_j^{1/2}\boldsymbol{\rho}_j\\ \boldsymbol{\rho}_j'\Sigma_j^{1/2} & 1\end{pmatrix}\right),$$

$$(\mathbf{y}_{ji}, x_i^*) \sim \mathcal{N}_{T+1}\left(\begin{pmatrix}\mu(\mathbf{y}_{ji})\\ \mu(x_i^*)\end{pmatrix}, \begin{pmatrix}\Omega_j & \lambda_x\boldsymbol{\lambda}_j\\ \lambda_x\boldsymbol{\lambda}_j' & 1+\lambda_x^2\end{pmatrix}\right),$$

in the SR and the SF model, respectively.

The observed treatment x_i restricts the range of the latent utility to either $I_0 = (-\infty, 0]$ or $I_1 = [0, +\infty)$, so that even if the latent utilities were available, $(\mathbf{y}_{ji}, x_i^*)$ were not observed in the full support $\Re^{T+1}$ but only in the restricted support $\Re^T \times I_0$ and $\Re^T \times I_1$, respectively, for $j = 0, 1$.

As the distribution of the latent utility conditional on the potential outcome is $x_i^*|\mathbf{y}_{ji} \sim \mathcal{N}\left(m_{ji}, s_j^2\right)$ with moments

$$m_{ji} = \mu(x_i^*) + \boldsymbol{\rho}_j'\Sigma_j^{1/2}\Omega_j^{-1}(\mathbf{y}_{ji} - \mu(\mathbf{y}_{ji})), \qquad s_j^2 = 1 - \boldsymbol{\rho}_j'\Sigma_j^{1/2}\Omega_j^{-1}\Sigma_j^{1/2}\boldsymbol{\rho}_j, \qquad \text{(SR)}$$

$$m_{ji} = \mu(x_i^*) + \lambda_x\boldsymbol{\lambda}_j'\Omega_j^{-1}(\mathbf{y}_{ji} - \mu(\mathbf{y}_{ji})), \qquad s_j^2 = 1 + \lambda_x^2(1 - \boldsymbol{\lambda}_j'\Omega_j^{-1}\boldsymbol{\lambda}_j), \qquad \text{(SF)}$$

the joint distributions of observed earnings sequence and treatment are given as

$$p(\mathbf{y}_{0i}, x_i = 0) = p(\mathbf{y}_{0i})\int_{-\infty}^{0} p(x_i^*|\mathbf{y}_{0i})\,dx_i^* \;=\; p(\mathbf{y}_{0i})(1 - \Phi(m_{0i}/s_0)),$$

$$p(\mathbf{y}_{1i}, x_i = 1) = p(\mathbf{y}_{1i})\int_{0}^{\infty} p(x_i^*|\mathbf{y}_{1i})\,dx_i^* \;=\; p(\mathbf{y}_{1i})\Phi(m_{1i}/s_1),$$

where $\Phi(z)$ denotes the cdf of the standard normal distribution, and $p(\mathbf{y}_{ji})$ is equal to the marginal distribution of $\mathbf{y}_{ji}$, given by $\mathcal{N}_T\left(\mu(\mathbf{y}_{ji}), \Omega_j\right)$.

Which of these two joint distributions has generated the data depends on the realized treatment $x_i = j$, so that the observed data $(\mathbf{y}_i, x_i = j)$ for subject i comes from

$$p(\mathbf{y}_i, x_i = j) = p(\mathbf{y}_{0i}, x_i = 0)\, I(x_i = 0) \;+\; p(\mathbf{y}_{1i}, x_i = 1)\, I(x_i = 1). \qquad (3.14)$$

Hence, the distribution of the observed outcome vector $\mathbf{y}_i$ given treatment i for subject i follows as:

$$p(\mathbf{y}_i|x_i = j) = \begin{cases} p(\mathbf{y}_{0i})\frac{1-\Phi(m_{0i}/s_0)}{1-\Phi(\mu(x_i^*)/\sigma_x)}, & x_i = 0,\\ p(\mathbf{y}_{1i})\frac{\Phi(m_{1i}/s_1)}{\Phi(\mu(x_i^*)/\sigma_x)}, & x_i = 1,\end{cases} \qquad (3.15)$$

which obviously is not a multivariate normal distribution, as m_{ji} is a function of the potential outcome $\mathbf{y}_{ji}$.

3.5 Treatment Effects

Under our flexible specifications of the potential outcome models in the previous sections the difference in the potential outcome sequences is given by the $T \times 1$ vector

$$\Delta_i = \mathbf{y}_{1i} - \mathbf{y}_{0i} = \mathbf{1}_T\kappa + \mathbf{W}_i\boldsymbol{\theta} + (\boldsymbol{\varepsilon}_{1i} - \boldsymbol{\varepsilon}_{0i}).$$

In both modeling frameworks we can identify a sequence of causal Average Treatment Effects (ATE) based on the differences in the means of the potential outcome sequences

$$ATE(\mathbf{W}) = E(\Delta_i|\mathbf{W}) = \mathbf{1}_T\kappa + \mathbf{W}\boldsymbol{\theta}, \tag{3.16}$$

where κ captures a common (homogeneous) treatment effect and $\mathbf{W}\boldsymbol{\theta}$ a heterogeneous treatment effect depending on subjects' demographic characteristics such as whether a mother is a blue collar or a white collar worker. These two types may face very different penalties from time spent on leave and different dynamic patterns of the treatment effects over the panel periods. In the estimation section we discuss in further detail how we can estimate these effects and take into account the empirical distribution of the demographic factors in $\mathbf{W}$ in the computation of the ATE.

While the average treatment effect provides an estimate of the expected gain or loss from maternity leave of a "typical" mother from the sample controlling for any selection bias, it is likely to either overstate or understate the gains or losses of a long maternity leave for mothers who chose the short treatment (untreated) or the long treatment (treated). The average treatment effects on the treated (TT) and untreated (TU), defined as:

$$TT(\mathbf{W}, \mathbf{v}, z) = E(\Delta_i|\mathbf{W}, \mathbf{v}, z, x_i = 1) \quad \text{and} \quad TU(\mathbf{W}, \mathbf{v}, z) = E(\Delta_i|\mathbf{W}, \mathbf{v}, z, x_i = 0)\ ,$$

measure those effects taking into account differences across these mother groups both in terms of observable and unobservable characteristics. In the context of our problem these two effects might be quite different from the ATE if mothers choose the length based on some information regarding the expected penalty from the length of leave or their attitudes towards work versus child care efforts. For example, mothers in jobs with high career prospects might choose a short leave to avoid extensive loss of human capital that would have strong negative effects if they were to take a long leave.

In the shared factor model these two effects can be identified from the distribution of the difference in the potential outcome sequences of subjects, which is based on the joint distribution of the errors $(\boldsymbol{\varepsilon}_{1i}, \boldsymbol{\varepsilon}_{0i})$. The implied joint distribution of the latent utility x_i^* and Δ_i in the shared factor model is given as

$$\begin{pmatrix} x_i^* \\ \Delta_i \end{pmatrix} \sim \mathcal{N}_T\left(\begin{pmatrix} \mathbf{v}_i\boldsymbol{\alpha}_1 + z_i\alpha_2 \\ \mathbf{1}_T\kappa + \mathbf{W}_i\boldsymbol{\theta} \end{pmatrix}, \begin{pmatrix} 1+\lambda_x^2 & \lambda_x(\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_0)' \\ \lambda_x(\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_0) & (\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_0)(\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_0)' + \Sigma_1 + \Sigma_0 \end{pmatrix}\right),$$

and allows to derive, e.g. the average treatment effects on treated and untreated (Heckman et al., 2014) as

$$TT(\mathbf{W}, \mathbf{v}, z) = E(\Delta_i|x_i^* > 0, \mathbf{W}, \mathbf{v}, z) = ATE(\mathbf{W}) + \frac{\lambda_x(\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_0)'}{\sigma_x}\frac{\phi(-\mu(x_i^*)/\sigma_x)}{\Phi(\mu(x_i^*)/\sigma_x)}, \tag{3.17}$$

$$TU(\mathbf{W}, \mathbf{v}, z) = E(\Delta_i|x_i^* < 0, \mathbf{W}, \mathbf{v}, z) = ATE(\mathbf{W}) - \frac{\lambda_x(\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_0)'}{\sigma_x}\frac{\phi(-\mu(x_i^*)/\sigma_x)}{1-\Phi(\mu(x_i^*)/\sigma_x)}, \tag{3.18}$$

where $\phi(z)$ denotes the pdf of the standard normal distribution and the second term captures the additional effects on the unobservables. For abbreviation we will use ψ_{TT} and ψ_{TU} to denote these additional effects and write in short

$$TT(\mathbf{W}, \mathbf{v}, z) = ATE(\mathbf{W}) + \psi_{TT}(\mathbf{W}, \mathbf{v}, z), \qquad TU(\mathbf{W}, \mathbf{v}, z) = ATE(\mathbf{W}) + \psi_{TU}(\mathbf{W}, \mathbf{v}, z)$$

.

4 Bayesian Inference

A fully Bayesian inference is applied to estimate model parameters and treatment effects, both for the switching regression models as well as the shared factor model introduced in the previous section. We discuss the choice of prior distributions for all model parameters, followed by a discussion of performing posterior inference by means of Markov chain Monte Carlo (MCMC) methods given panel data as in our specific application. In Subsection 4.1, we specify a set of standard priors, which is extended in Subsection 4.2 to a more flexible set of prior distributions to implement variable selection in the context of both modeling frameworks. As described in Subsection 4.3, under both sets of prior specifications inference about model parameters and treatment effects can be done with standard MCMC methods.

To simplify the discussion throughout this section, we use a more compact notation and write the structural mean of the selection equation (3.1) as $\mu(x_i^*) = \mathbf{Z}_i\boldsymbol{\alpha}$, where $\mathbf{Z}_i = (\mathbf{v}_i, z_i)$ denotes the $1 \times d_\alpha$ covariate vector for subject i in the selection model and $\boldsymbol{\alpha} = (\boldsymbol{\alpha}_1, \alpha_2)$ is the corresponding vector of d_α regression coefficients. The structural mean of outcome under treatment j (equations (3.2) and (3.3)) is denoted by $\mu(\mathbf{y}_{ji}) = \mathbf{W}_{ji}\boldsymbol{\beta}$, where

$$\mathbf{W}_{ji} = \begin{cases} \begin{pmatrix} \mathbf{1}_T & \mathbf{W}_i & \mathbf{0}_T & \mathbf{O} \end{pmatrix}, & \text{for } j = 0, \\ \begin{pmatrix} \mathbf{1}_T & \mathbf{W}_i & \mathbf{1}_T & \mathbf{W}_i \end{pmatrix}, & \text{for } j = 1, \end{cases}$$

denotes the $T \times d_\beta$ covariate matrix for $\mathbf{y}_{ji}$ in the outcome model and $\boldsymbol{\beta} = (\mu, \boldsymbol{\gamma}, \kappa, \boldsymbol{\theta})$ is the corresponding vector of d_β regression coefficients.

4.1 Priors

Bayesian model specification is completed by assigning prior distributions to all model parameters. For each model $\mathcal{M}$ we use a prior $p^{\mathcal{M}}(\Theta^{\mathcal{M}})$, where $\Theta^{\mathcal{M}}$ denotes the collection of all parameters in model $\mathcal{M}$. Our priors are of the following structure

$$p^{SRI}(\Theta^{SRI}) = p(\boldsymbol{\beta})p(\boldsymbol{\alpha}) \prod_{j=0}^{1} p^{SR}(\boldsymbol{\sigma}_j)p^{SR}(\boldsymbol{\rho}_j)p^{SRI}(D_j), \tag{4.1}$$

$$p^{SRF}(\Theta^{SRF}) = p(\boldsymbol{\beta})p(\boldsymbol{\alpha}) \prod_{j=0}^{1} p^{SR}(\boldsymbol{\sigma}_j)p^{SR}(\boldsymbol{\rho}_j)p^{SRF}(\boldsymbol{\lambda}_j), \tag{4.2}$$

$$p^{SF}(\Theta^{SF}) = p(\boldsymbol{\beta})p(\boldsymbol{\alpha})\, p^{SF}(\lambda_x) \prod_{j=0}^{1} p^{SF}(\boldsymbol{\sigma}_j)p^{SF}(\boldsymbol{\lambda}_j). \tag{4.3}$$

In all three models standard priors for the regression parameters $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are normal distributions $\boldsymbol{\alpha} \sim \mathcal{N}_{d_\alpha}(\mathbf{a}_0, \mathbf{A}_0)$ and $\boldsymbol{\beta} \sim \mathcal{N}_{d_\beta}(\mathbf{b}_0, \mathbf{B}_0)$. To incorporate variable selection we use spike and slab prior distributions, which will be described in section 4.2.

In the shared factor model the property that x_i^* and $\mathbf{y}_{ji}$ are independent conditioning on the latent factor f_i allows to specify conditionally conjugate priors for all parameters. We use independent inverse Gamma-priors for the idiosyncratic variances

$$\sigma_{j,t}^2 \sim \mathcal{G}^{-1}(s_{0,jt}, S_{0,jt}),$$

and normal priors for the factor loadings $\lambda_x \sim \mathcal{N}(l_{x0}, L_{x0})$ and $\boldsymbol{\lambda}_j \sim \mathcal{N}_T(\mathbf{l}_{j0}, \mathbf{L}_{j0})$.

In the switching regression models, prior specification for idiosyncratic variances and correlations is more involved. From the lower triangular Cholesky factor $\mathbf{G}_j$ of $\mathsf{Cov}(\boldsymbol{\epsilon}_{ji}, \eta_i)$ which is given as

$$\mathbf{G}_j = \begin{pmatrix} \Sigma_j^{1/2} & \mathbf{0}_T \\ \boldsymbol{\rho}_j' & (1 - \sum_{t=1}^{T} \rho_{j,t}^2)^{1/2} \end{pmatrix},$$

it is obvious that positive-definiteness of $\mathbf{G}_j$ and hence $\mathsf{Cov}(\boldsymbol{\epsilon}_{ji}, \eta_i)$ is guaranteed if $(1 - \sum_{t=1}^{T} \rho_{j,t}^2) > 0$. Following Chib and Jacobi (2007), we assign to $\boldsymbol{\rho}_j$ a T-variate Normal prior $\mathcal{N}_T(\mathbf{r}_0, \mathbf{R}_0)$ truncated to the region yielding a positive definite Cholesky factor $\mathbf{G}_j$ and to $\ln \boldsymbol{\sigma}_j$ a T-variate Normal distribution $\mathcal{N}_T(\mathbf{c}_{0j}, \mathbf{C}_{0j})$. Finally, we specify conditionally conjugate priors for the random intercept variances, $D_j \sim \mathcal{G}^{-1}(d_{j0}, D_{j0})$, in the SRI model as well as for the factor loadings, $\boldsymbol{\lambda}_j \sim \mathcal{N}_T(\mathbf{l}_{j0}, \mathbf{L}_{j0})$, in the SRF model.

4.2 Variable Selection with spike and slab priors

The prior choices specified above assume that all covariates in $\mathbf{Z}_i$ and $\mathbf{W}_{ji}$ are included in the selection and potential earnings models, respectively. In the latter this very general specification implies that the effects of all covariates in the outcome model in matrix $\mathbf{W}_{ji}$ vary by treatment as captured by $\boldsymbol{\theta}$, i.e. heterogeneous treatment effects, in addition to the presence of a constant treatment effect captured by κ. This general model might be overspecified.

We now introduce variable selection to decide which covariates should be included in the regression models for treatment selection, i.e. in the probit model for selection of treatment x_i, and which covariates in the models for the potential outcomes. In the latter, variable selection will also enable us to test for the presence of heterogeneous treatment effects captured by $\boldsymbol{\theta}$, as we have specified the models very generally to allow for covariate effects which differ by treatment, but this general model might also be overspecified.

In a Bayesian approach, selection of relevant regressors can be performed by specifying spike and slab prior distributions for the corresponding regression effects captured by $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$. These prior distributions are mixtures of two components, a spike at zero to shrink small coefficients to zero and a flat slab component, to leave large effects (nearly) unshrunk. Spike and slab prior distribution have been widely used in different variants in regression type models, e.g. Mitchell and Beauchamp (1988); George and McCulloch (1997); Geweke (1996); Ishwaran and Rao (2005) but also for more general model selection problems (see e.g. Frühwirth-Schnatter and Tüchler (2008); Frühwirth-Schnatter and Wagner (2010)).

We will use spike and slab prior distributions for the regression effects $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ in all models. To this aim, we introduce for each regression effect a binary indicator which takes the value 1, if the effect is assigned to the slab component, and zero otherwise. We assume conditional independence of the regression effects assigned to the slab component, but other choices are also possible.

The prior for $\boldsymbol{\alpha}$ is specified as

$$p(\boldsymbol{\alpha}|\boldsymbol{\nu}) = \prod_{j:\nu_j=1} p_{\text{slab}}(\alpha_j) \prod_{j:\nu_j=0} p_{\text{spike}}(\alpha_j), \tag{4.4}$$

where $\boldsymbol{\nu} = (\nu_1, \ldots, \nu_{d_\alpha})$ is a vector of binary indicators. The indicators are assumed independent with $p(\nu_j) = \pi_\alpha$ and π_α is assigned a uniform hyper-prior $\pi_\alpha \sim \mathcal{U}[0,1]$. We use a Dirac spike, i.e. a point mass at zero, $p_{\text{spike}}(\alpha_j) = \Delta_0(\alpha_j)$ and a normal slab $p_{\text{slab}}(\alpha_j) = p(\alpha_j|\mathcal{N}(0, A_0))$.

Similarly, we define a further vector of binary indicators $\boldsymbol{\delta} = (\delta_1, \ldots, \delta_{d_\beta})$ and specify the prior for $\boldsymbol{\beta}$ as

$$p(\boldsymbol{\beta}|\boldsymbol{\delta}) = \prod_{j:\delta_j=1} p(\beta_j|\mathcal{N}(0, B_0)) \prod_{j:\delta_j=0} \Delta_0(\beta_j), \tag{4.5}$$

where the components of $\boldsymbol{\delta}$ are independent with $p(\delta_j) = \pi_\beta$ and $\pi_\beta \sim \mathcal{U}[0,1]$.

In the shared factor model the factor loadings $\boldsymbol{\lambda}_j$ can be interpreted as regression effects of the latent factor. Hence it is straightforward to perform selection also for the factor loadings $\boldsymbol{\lambda} = (\boldsymbol{\lambda}_0, \boldsymbol{\lambda}_1)$ by assigning a spike and slab prior distribution. We specify the prior for $\boldsymbol{\lambda}$ as

$$p^{SF}(\boldsymbol{\lambda}|\boldsymbol{\delta}^\lambda) = \prod_{k:\delta_k^\lambda=1} p(\lambda_k|\mathcal{N}(0, L_0)) \prod_{k:\delta_k^\lambda=0} \Delta_0(\lambda_k)$$

where $\boldsymbol{\delta}^\lambda$ is the corresponding $2T \times 1$ vector of binary indicators with $p(\delta_j^\lambda = 1) = \pi_\lambda$. The prior inclusion probability of the factor loadings π_λ is assigned a uniform hyper-prior $\pi_\lambda \sim \mathcal{U}[0,1]$.

4.3 Model Estimation

We subsume now in $\Theta^\mathcal{M}$all parameters of model $\mathcal{M}$, i.e. if variable selection is performed $\Theta^\mathcal{M}$ includes also the hyper-parameters $\boldsymbol{\nu}, \boldsymbol{\delta}, \pi_\alpha$ and π_β. The goal is posterior inference for $\Theta^\mathcal{M}$ based on the respective posterior distribution, which is proportional to likelihood times prior,

$$p(\Theta^\mathcal{M}|\mathbf{x}, \mathbf{y}) \propto p^\mathcal{M}(\Theta^\mathcal{M}) \prod_{i=1}^n p(\mathbf{y}_i, x_i|\Theta^\mathcal{M}). \tag{4.6}$$

The prior distributions for the SR and SF model are defined in the previous sections and the likelihood contributions for the two models are given in equation (3.14).

Given the different model specifications, we employ different MCMC sampling schemes for the switching regression and the shared factor model. However, for both models we augment the parameter space to improve the tractability of the posterior distribution. As standard, the MCMC implementation relies on the latent utility specification of the probit model (Albert and Chib, 1993) and the likelihood contribution of observed outcome and treatment intake, augmented by the latent utility, is given as

$$p(x_i^*, \mathbf{y}_i, x_i = j) = p(x_i^*, \mathbf{y}_{ji}|x_i = j)p(x_i = j).$$

We emphasize here that though $p(x_i^*, \mathbf{y}_{ji}|x_i = j)$ is the likelihood of a $(T+1)$-dimensional truncated normal distribution,

$$p(x_i^*, \mathbf{y}_{ji}|x_i = j) = \begin{cases} p(x_i^*, \mathbf{y}_{0i})/P(x_i^* \leq 0), & \text{if } x_i = 0, \\ p(x_i^*, \mathbf{y}_{1i})/P(x_i^* > 0), & \text{if } x_i = 1, \end{cases}$$

the total likelihood contribution of subject i is identical with the likelihood contribution from the joint $(T+1)$-variate normal distribution of $(x_i^*, \mathbf{y}_{ji})$.

To improve the structure of the likelihood and the posterior for a simulation of the posterior by standard MCMC methods we further augment the parameter space not only with the latent utilities $\{x_i^*\}$ but also with the latent variables $\mathbf{b}$ in the SR and the latent factors $\mathbf{f} = (f_1, \dots, f_n)$ in the SF model.

4.3.1 Posterior Inference for the Switching Regression Model

The basis for posterior inference is the augmented posterior distribution

$$p(\Theta^{SR}, \mathbf{x}^*, \mathbf{b}|\mathbf{y}, \mathbf{x}) \propto p^{SR}(\Theta^{SR})\, p(\mathbf{b}) \prod_{i=1}^n p(x_i^*, \mathbf{y}_i, x_i|\Theta^{SR}, \mathbf{b}). \tag{4.7}$$

where $\mathbf{x}^* = (x_1^*, \dots, x_n^*)$. For data augmentation with the latent utilities x_i^*, these are sampled from their full conditional distribution. From the joint error distribution given in equation (3.8) we derive

$$x_i^*|x_i = j, \mathbf{y}_{ji}, \mathbf{b}, \Theta^{SR} \sim \mathcal{N}\left(\mathbf{Z}_i\boldsymbol{\alpha} + \boldsymbol{\omega}_j'\Sigma_j^{-1}\boldsymbol{\epsilon}_{ji}, 1 - \boldsymbol{\omega}_j'\Sigma_j^{-1}\boldsymbol{\omega}_j\right), \tag{4.8}$$

where $\boldsymbol{\epsilon}_{ji} = \mathbf{y}_{ji} - \mathbf{W}_{ji}\boldsymbol{\beta} - \mathbf{1}_T b_{ji}$ in the SRI and $\boldsymbol{\epsilon}_{ji} = \mathbf{y}_{ji} - \mathbf{W}_{ji}\boldsymbol{\beta} - \boldsymbol{\lambda}_j\tilde{b}_{ji}$ in the SRF model. Conditioning further on x_i truncates this normal distribution to the interval I_{x_i}.

We sample the indicators $(\boldsymbol{\nu}, \boldsymbol{\delta})$, the regression effects $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ and the random intercepts (or factors) $\mathbf{b}$ in one block. Marginalizing over the random intercepts (factors), $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ is the vector of regression effects in a multivariate linear normal regression model for $(x_i^*, \mathbf{y}_{x_i,i})$, where variable selection is a standard step. Conditional on the rest of the parameters, also sampling of random intercepts (in the SRI model) or latent factors (in the SRF model) is a standard Gibbs draw. As discussed in section 3.3.2, for the SRF model all elements of the covariance matrix Ω_j are fully identified though the signs of the factor loadings and the factors $\mathbf{b}$ are not separately identified. To guarantee sampling from the whole range of the posterior this non-identifiability is taken into account in posterior sampling by performing a random sign-switch of $\mathbf{b}$ and $\boldsymbol{\lambda}_j$.

Finally, the full conditionals of the remaining parameters $\boldsymbol{\sigma}_0, \boldsymbol{\sigma}_1, \boldsymbol{\rho}_0, \boldsymbol{\rho}_1$, given as

$$p(\boldsymbol{\sigma}_j|\boldsymbol{\alpha}, \boldsymbol{\beta}, \mathbf{b}, \mathbf{y}, \mathbf{x}^*, \mathbf{x}) \propto p^{SR}(\boldsymbol{\sigma}_j) \prod_{i:x_i=j} p(\mathbf{y}_{ji}, x_i^*|\boldsymbol{\sigma}_j, \boldsymbol{\rho}_j, \boldsymbol{\alpha}, \boldsymbol{\beta}, \mathbf{b}, x_i), \tag{4.9}$$

$$p(\boldsymbol{\rho}_j|\boldsymbol{\alpha}, \boldsymbol{\beta}, \mathbf{b}, \mathbf{y}, \mathbf{x}^*, \mathbf{x}) \propto p^{SR}(\boldsymbol{\rho}_j) \prod_{i:x_i=j} p(\mathbf{y}_{ji}, x_i^*|\boldsymbol{\rho}_j, \boldsymbol{\sigma}_j, \boldsymbol{\alpha}, \boldsymbol{\beta}, \mathbf{b}, x_i), \tag{4.10}$$

are not of closed form and hence these parameters are sampled from their full conditionals using an Metropolis-Hastings (MH)-step. Full details of all sampling steps are given in Appendix A.1.

4.3.2 Posterior Inference for the Shared Factor Model

The basis for the posterior inference is the augmented posterior distribution

$$p(\Theta^{SF}, \mathbf{x}^*, \mathbf{f}|\mathbf{y}, \mathbf{x}) \propto p^{SF}(\Theta^{SF})\, p(\mathbf{f})\, p(\mathbf{x}^*, \mathbf{y}, \mathbf{x}|\Theta^{SF}, \mathbf{f}). \tag{4.11}$$

Conditional on the latent factor the likelihood of the joint model is given as

$$p(\mathbf{y}, \mathbf{x}, \mathbf{x}^*|\Theta^{SF}, \mathbf{f}) = p(\mathbf{y}|\boldsymbol{\beta}, \boldsymbol{\sigma}, \boldsymbol{\lambda}_0, \boldsymbol{\lambda}_1, \mathbf{f})\; p(\mathbf{x}, \mathbf{x}^*|\boldsymbol{\alpha}, \lambda_x, \mathbf{f}),$$

which is the product of the likelihood contributions from two standard normal regression models, where all parameters can be drawn directly from their full conditional posterior distributions under conjugate priors. From the specification of the error terms in these models in equations (3.9) to (3.11), the latent factors f_i have a normal posterior distribution $\mathcal{N}(f_{n,i}, F_{n,i})$ with the posterior moments depending on $x_i = j$:

$$F_{n,i} = 1/(1 + \lambda_x^2 + \sum_{t=1}^{T} \frac{\lambda_{j,t}^2}{\sigma_{j,t}^2}), \quad f_{n,i} = (\lambda_x, \frac{\lambda_{j,1}}{\sigma_{j,1}}, \dots, \frac{\lambda_{j,T}}{\sigma_{j,T}}) \begin{pmatrix} x_i^* - \mathbf{Z}_i\boldsymbol{\alpha} \\ \mathbf{y}_{j,i} - \mathbf{W}_{ji}\boldsymbol{\beta} \end{pmatrix}. \tag{4.12}$$

The non-identifiability of the signs of the latent factors and the factor loadings (discussed in Section 3.3.2) is taken into account in the sampling scheme via a random sign-switch.

The detailed steps of the sampler are provided in Appendix A.2. The sampling scheme does not rely on imputation of the unobserved potential outcome, although this would be possible based on the joint distribution of both potential outcome vectors. However, while we observed that such a sampler provides essentially the same results it is slower and less efficient with respect to autocorrelation of the draws.

4.4 Treatment Effects Estimation

4.4.1 Average treatment effects

The average treatment effect for a given covariate vector $\mathbf{W}$ can be estimated based on the posterior distribution of the $ATE(\mathbf{W})$ which is obtained by integrating out the model parameters Θ with respect to the posterior distribution as

$$p(ATE|\mathbf{W}, \mathbf{x}, \mathbf{y}) = \int (\mathbf{1}_T \kappa + \mathbf{W}\boldsymbol{\theta}) p(\Theta|\mathbf{x}, \mathbf{y}) d\Theta.$$

An estimate of $ATE(\mathbf{W})$ in terms of the mean of the posterior distribution is obtained as

$$\widehat{ATE}(\mathbf{W}) = \mathbf{1}_T \hat{\kappa} + \mathbf{W}\hat{\boldsymbol{\theta}},$$

where the posterior mean estimates are computed from MCMC draws as $\hat{\kappa} = \frac{1}{M}\sum_{m=1}^{M} \kappa^{(m)}$ and $\hat{\boldsymbol{\theta}} = \frac{1}{M}\sum_{m=1}^{M} \boldsymbol{\theta}^{(m)}$.

By integration over the covariates with respect to the empirical distribution of the data we obtain the insample average treatment effect. For panel outcomes we are interested in the evolvement of the insample average treatment effect. To capture the dynamics of the insample average treatment effect, which is not attributable to changes in covariates, we define the insample average treatment effect at panel time point t, $ATE(t)$, as the mean with respect to the distribution of the demographic covariates at time point $t = 1$, i.e. the average treatment effect at time t for the mothers observed in the sample with their covariate values at $t = 1$. $ATE(t)$ can be estimated by

$$\widehat{ATE}(t) = \hat{\kappa} + \frac{1}{n}\sum_{i=1}^{n} \tilde{\mathbf{W}}_{i,1}(t)\, \hat{\boldsymbol{\theta}},$$

where $\tilde{\mathbf{W}}_{i,1}(t)$ denotes the covariate vector of subject i with panel dummies for time point t and values of all other covariates (except panel dummies) observed at $t = 1$.

4.4.2 Treatment effects on treated and untreated

As discussed in Section 3.5 treatment effects of treated and untreated are available only from the joint distribution of potential outcomes, which is not specified in the switching regression models, but implied by the shared factor in the SF model. Similarly to the average treatment effect the posterior distribution of the treatment effect of the treated and untreated given in equations (3.17) and (3.18) can be expressed as

$$p(TT|\mathbf{W}, \mathbf{Z}, \mathbf{x}, \mathbf{y}) = \int \Big(\mathbf{1}_T \kappa + \mathbf{W}\boldsymbol{\theta} + \psi_{TT}(\Theta)\Big) p(\Theta|\mathbf{x}, \mathbf{y}) d\Theta,$$

$$p(TU|\mathbf{W},\mathbf{Z},\mathbf{x},\mathbf{y}) = \int \Big(\mathbf{1}_T\kappa + \mathbf{W}\boldsymbol{\theta} + \psi_{TU}(\Theta)\big)p(\Theta|\mathbf{x},\mathbf{y})d\Theta,$$

where the model parameters are integrated out with respect to their posterior distribution. We estimate $TT(\mathbf{W},\mathbf{Z})$ and $TU(\mathbf{W},\mathbf{Z})$ by the mean of the posterior distribution, which is approximated from the MCMC draws as

$$\widehat{TT}(\mathbf{W},\mathbf{Z}) = \widehat{ATE}(\mathbf{W}) + \frac{1}{M}\sum_{m=1}^{M}\psi_{TT}(\Theta^{(m)}),$$
$$\widehat{TU}(\mathbf{W},\mathbf{Z}) = \widehat{ATE}(\mathbf{W}) + \frac{1}{M}\sum_{m=1}^{M}\psi_{TU}(\Theta^{(m)}).$$

To estimate in-sample treatment effects on treated and untreated we integrate over the empirical distribution of covariates. Corresponding to the treatment effect at panel time point t, $ATE(t)$ defined above we estimate the insample treatment effect on treated and untreated at panel time point t as

$$\widehat{TT}(t) = \frac{1}{n_1}\sum_{i:x_i=1}\widehat{TT}(\tilde{\mathbf{W}}_{i,1}(t),\mathbf{Z}_i),$$
$$\widehat{TU}(t) = \frac{1}{n_0}\sum_{i:x_i=0}\widehat{TT}(\tilde{\mathbf{W}}_{i,1}(t),\mathbf{Z}_i).$$

Here n_j, $j = 0, 1$ is the number of subjects with $x_i = j$ observed at $t = 1$.

5 Simulation study

We have performed a small simulation study to test the performance of the MCMC samplers and to explore consequences of a mis-specification of the covariance structure of the panel outcomes or dependence between outcome and latent utility. The details of the simulation design described below were chosen to help illustrate the effect of the mis-specification of the dependence structure on the estimation results.

5.1 Simulation Setup

We have generated three data sets of $n = 50000$ subjects with $T = 4$ panel periods from each of the three models models specified in Section 3 (data 1: SRI, data 2: SRF and data 3: SF). In each case the structural mean of the latent utility (equation (3.1)) is specified as

$$\mu(x_i^*) = \mathbf{Z}_i\boldsymbol{\alpha} = \alpha_{10} + \alpha_{11}v_{1i} + \alpha_{12}v_{2i} + \alpha_2 z_i$$

with $\mathbf{Z}_i = (1, v_{1i}, v_{2i}, z_i)$, where v_{1i} is standard normal and v_{2i} and z_i are binary variables with $p(v_{2i} = 1) = p(z_i = 1) = 0.5$, and $\alpha = (\boldsymbol{\alpha}_1, \alpha_2) = (-0.9, 0.8, 0, 1.5)$. To generate the outcome sequences (equations (3.2) - (3.3)) we used a linear predictor with v_{1i} and v_{2i} and dummies for the panel time points $t = 2, 3, 4$ as regressors. The common intercept and covariate effects were set at $(\mu, \boldsymbol{\gamma}) = (3, 1, 0, 0.1, 0.15, 0.2)$, and the constant and heterogeneous treatment effects of the covariates at $(\kappa, \boldsymbol{\theta}) = (-0.5, 0, 0.2, -0.1, 0, 0.1)$. The implied average treatment effects for the four panel periods are $(-0.4, -0.5, -0.4, -0.3)$. Error variances were set to $\boldsymbol{\sigma}_0^2 = 0.25\,\mathbf{1}_4$ and $\boldsymbol{\sigma}_1^2 = \mathbf{1}_4$.

For the two data sets generated from the switching regression models we set the correlations at $\boldsymbol{\rho}_0 = (0.6, 0.5, 0.4, 0.3)$ and $\boldsymbol{\rho}_1 = -\boldsymbol{\rho}_0$ to capture a dependence between the latent utility and the potential outcomes that varies over time. The covariance structures for the potential outcomes were defined by setting the random intercept variances to $D_0 = 0.4$ and $D_1 = 0.8$ for data set 1 under the SRI, and by setting the factor loadings to $\boldsymbol{\lambda}_0 = (0.4, 0.35, 0.3, 0.25)$ and $\boldsymbol{\lambda}_1 = (0.7, 0.6, 0.5, 0.4)$ for data set 2 under the SRF. For data set 3, generated under the SF model, we set $\lambda_x = 0.7$ and $\boldsymbol{\lambda}_0 = (0.6, 0.6, 0.5, 0.5)$ and $\boldsymbol{\lambda}_1 = -\boldsymbol{\lambda}_0$. The settings for $\boldsymbol{\lambda}_0$ and $\boldsymbol{\lambda}_1$ imply a full covariance matrix for the potential outcomes with the covariances across the potential outcomes varying over time under SRF and SF models, compared to the more restrictive compound symmetry structure under the SRI. The implied comparable marginal correlations between latent utility and potential outcomes in data sets 1 to 3, marginalized over the latent factor or the random intercepts respectively, are

$$\begin{aligned}
&\mathsf{Cor}(x_i^*, \mathbf{y}_{0i}) = (0.37, 0.31, 0.25, 0.29) \qquad && \mathsf{Cor}(x_i^*, \mathbf{y}_{1i}) = (-0.44, -0.37, -0.30, -0.22)\\
&\mathsf{Cor}(x_i^*, \mathbf{y}_{0i}) = (0.47, 0.41, 0.34, 0.27) \qquad && \mathsf{Cor}(x_i^*, \mathbf{y}_{1i}) = (-0.49, -0.43, -0.36, -0.28)\\
&\mathsf{Cor}(x_i^*, \mathbf{y}_{0i}) = (0.44, 0.44, 0.40, 0.40) \qquad && \mathsf{Cor}(x_i^*, \mathbf{y}_{1i}) = (-0.30, -0.30, -0.26, -0.26).
\end{aligned}$$

For each data set Bayesian inference was carried out under each of the three model specifications with variable selection on the regression effects. The reported results are based on 10,000 iterations, following 10,000 burn-in iterations of the corresponding MCMC algorithm described in Section 4. For a faster convergence of the sampler, the first 5000 burn-in iterations are drawn from the full model which enables the MCMC chain to reach regions of higher posterior density without the additional computational burden of variable selection. As common in the variable selection literature we do not perform variable selection on the intercept.

5.1.1 Results for Regression Effects

In Table 1 we report the estimates of the regression effects in the potential outcome models for all three data sets (row blocks) under the SRI, SRF and SF models (column blocks). The diagonal cell blocks in the table refer to the estimates when the inference is based on the correct model. Bold numbers indicate biased estimates where the true value is not contained in the 99% posterior density interval. A star indicates that the inclusion probability is estimated above 0.5, suggesting that the covariate should be included in the model.

The results show that the true parameters are recovered well when the correct model is applied for the inference. However, when the inference is based on an incorrect model specification (off-diagonal cell blocks), the estimated regression effects in the potential outcome models are partially effected by model mis-specification. As shown in the next section this is due to the different assumptions about the dependence structures across the three models that result in biased estimates of the dependence parameters.

Here we observe biased estimates of the intercept μ, the constant treatment effect κ and the modified effects of v_1 and panel time. An exception is the SRF model for data set 1. Being more general than the SRI model, for the data set 1 it yields almost identical results to the SRI model, whereas for data set 2 some panel effects and their modifications are biased under the SRI model. Further, for both data sets 1 and 2 estimates of intercept modification and panel time effects are biased when employing the SF model. Finally, for data set 3 both the SRI and

Table 1: Results outcome equation: posterior means, sd (in parentheses) of regression effects

	SRI Model		SRF Model		SF Model	
	$(\mu, \boldsymbol{\gamma})$	$(\kappa, \boldsymbol{\theta})$	$(\mu, \boldsymbol{\gamma})$	$(\kappa, \boldsymbol{\theta})$	$(\mu, \boldsymbol{\gamma})$	$(\kappa, \boldsymbol{\theta})$
data 1						
μ, κ	2.998 (0.007)	-0.505 (0.015)*	2.997 (0.007)	-0.501 (0.016)*	2.991 (0.006)	**-0.617** (0.013)*
v_1	1.007 (0.004)*	-0.000 (0.001)	1.007 (0.004)*	0.000 (0.001)	**1.019** (0.004)*	0.000 (0.003)
v_2	0.000 (0.002)	0.186 (0.013)*	0.000 (0.002)	0.185 (0.013)*	0.000 (0.002)	0.185 (0.013)*
$t = 2$	0.104 (0.006)*	-0.098 (0.013)*	0.105 (0.006)*	-0.098 (0.014)*	**0.124** (0.004)*	**-0.060** (0.011)*
$t = 3$	0.163 (0.006)*	0.000 (0.002)	0.163 (0.006)*	0.000 (0.003)	**0.203** (0.004)*	**0.073** (0.011)*
$t = 4$	0.216 (0.006)*	0.105 (0.013)*	0.218 (0.006)*	0.099 (0.014)*	**0.280** (0.004)*	**0.198** (0.011)*
data 2						
μ, κ	**2.966** (0.006)	-0.515 (0.013)*	2.998 (0.006)	-0.503 (0.012)*	3.012 (0.005)	**-0.557** (0.009)*
v_1	0.998 (0.003)*	0.000 (0.001)	0.998 (0.003)*	-0.000 (0.001)	1.006 (0.002)*	0.000 (0.000)
v_2	0.000 (0.001)	0.202 (0.009)*	0.000 (0.000)	0.201 (0.009)*	0.000 (0.001)	0.201 (0.009)*
$t = 2$	**0.127** (0.007)*	-0.106 (0.015)*	0.110 (0.006)*	-0.096 (0.014)*	0.108 (0.005)*	-0.090 (0.011)*
$t = 3$	**0.196** (0.006)*	0.004 (0.012)	0.161 (0.006)*	0.000 (0.003)	0.158 (0.005)*	0.000 (0.002)
$t = 4$	**0.249** (0.006)*	**0.145** (0.014)*	0.194 (0.006)*	0.118 (0.014)*	0.208 (0.005)*	**0.145** (0.011)*
data 3						
μ, κ	**2.942** (0.006)	-0.476 (0.017)*	**2.959** (0.006)	-0.476 (0.017)*	2.997 (0.007)	-0.513 (0.010)*
v_1	**0.987** (0.005)*	0.010 (0.012)	**0.984** (0.005)*	0.017 (0.013)*	1.000 (0.003)*	-0.000(0.002)
v_2	0.000 (0.001)	0.195 (0.009)*	0.000 (0.001)	0.195 (0.009)*	0.000 (0.001)	0.195 (0.009)*
$t = 2$	0.090 (0.007)*	-0.064 (0.017)*	0.095 (0.005)*	-0.072 (0.015)*	0.108 (0.005)*	-0.093 (0.010)*
$t = 3$	**0.198** (0.006)*	-0.001 (0.006)	0.157 (0.006)*	-0.000 (0.003)	0.153 (0.004)*	-0.000 (0.002)
$t = 4$	**0.240** (0.006)*	0.123 (0.015)*	0.201 (0.007)*	0.117 (0.016)*	0.205 (0.005)*	0.105 (0.011)*

the SRF model yield biased estimates of the intercept μ, as well as the effect of variable v_1. Mis-specification may affect correct selection of variables as a result of biased estimates. For example, in data set 1 the SF model incorrectly selects the effect modification for $t = 3$, while in data set 3 the effect modification of v_1 is incorrectly selected under the SRF model with the inclusion probability estimated at 0.692. Under the SRI model the smaller bias results in inclusion probability just below 0.5.

In the case of the selection model, the estimates for the regression effects are almost identical across three models for each data set, suggesting that the estimation of the parameters as well as variable selection in the selection equation is not affected by the mis-specification (see Table 12 in Appendix B).

5.1.2 Results for Dependence Structures

The main differences between the three models are their assumptions with respect to the dependence structure within the latent outcome vectors and between outcomes and latent utility. We recall that the assumptions of the switching regression and shared factor models with respect to the dependence between the latent utility and potential outcomes are rather contrary: in the shared factor model all correlation between latent utility and potential outcomes is attributed to the latent factor which also determines the dependence structure within the potential outcome vectors. In the switching regression models, the latent variables only capture the dependence within the potential outcomes, whereas dependence between the error terms of the potential outcome vector and the latent utility is captured by the correlations $\boldsymbol{\rho}_j$. It is therefore more suitable to focus on the marginal correlations between latent utility and the panel outcomes (marginalizing over latent factor or random intercept) when comparing the estimated correlations across the three model specifications. For the shared factor model, this correlation is given in equation (3.13) and for the switching regression models the correlation marginalized over the random intercept and the latent factor is given as

$$\text{SRI: } \mathsf{Cor}(y_{j,it}, x_i^*) = \frac{\omega_{j,t}}{\sqrt{\sigma_{j,t}^2 + D_j}}, \tag{5.1}$$

$$\text{SRF: } \mathsf{Cor}(y_{j,it}, x_i^*) = \frac{\omega_{j,t}}{\sqrt{\sigma_{j,t}^2 + \lambda_{j,t}^2}}, \tag{5.2}$$

where the numerators are from the diagonal of the marginalized covariance matrix Ω_j of the potential outcomes. In Table 2 we report the differences between estimated and the true correlations. Estimates, where the true, data generating value is not included in the 99% posterior interval are given in bold.

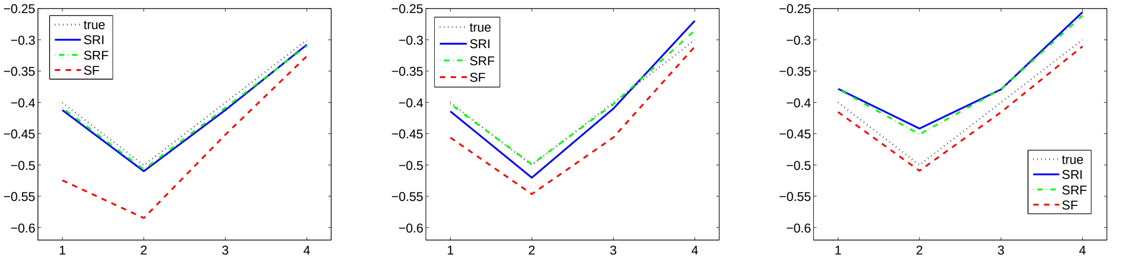
Table 2: Marginal correlation between latent utility and outcome: difference between posterior means and true values

		SRI Model		SRF Model		SF Model	
data	t	treatment 0	treatment 1	treatment 0	treatment 1	treatment 0	treatment 1
1	1	-0.001 (0.011)	0.016 (0.011)	-0.002 (0.009)	0.013 (0.011)	**-0.029** (0.008)	**0.159** (0.007)
	2	0.011 (0.011)	0.012 (0.011)	0.011 (0.012)	0.009 (0.013)	**0.031** (0.008)	**0.084** (0.007)
	3	0.024 (0.011)	0.011 (0.011)	0.024 (0.011)	0.003 (0.013)	**0.096** (0.008)	0.008 (0.007)
	4	0.028 (0.012)	-0.008 (0.013)	0.029 (0.012)	-0.006 (0.015)	**0.158** (0.008)	**-0.065** (0.007)
2	1	**-0.103** (0.013)	**0.064** (0.011)	-0.021 (0.011)	0.007 (0.010)	0.013 (0.008)	**0.057** (0.008)
	2	**-0.034** (0.013)	**0.040** (0.012)	0.007 (0.012)	-0.007 (0.012)	**0.034** (0.007)	**0.039** (0.008)
	3	0.032 (0.013)	-0.020 (0.012)	0.022 (0.012)	-0.018 (0.011)	**0.050** (0.007)	**0.035** (0.008)
	4	**0.057** (0.014)	**-0.056** (0.013)	-0.019 (0.013)	0.001 (0.013)	**0.068** (0.007)	-0.008 (0.008)
3	1	**-0.107** (0.008)	**0.037** (0.017)	**-0.075** (0.007)	0.017 (0.168)	-0.009 (0.009)	0.012 (0.008)
	2	**-0.147** (0.008)	0.014 (0.017)	**-0.103** (0.005)	-0.004 (0.182)	-0.007 (0.009)	0.007 (0.008)
	3	**-0.026** (0.006)	-0.034 (0.018)	**-0.074** (0.006)	-0.007 (0.170)	-0.008 (0.009)	0.001 (0.007)
	4	**-0.054** (0.006)	-0.033 (0.019)	**-0.095** (0.010)	0.001 (0.189)	-0.009 (0.009)	0.012 (0.007)

As expected, estimates correspond well to the true values when the appropriate model is used for the analysis. Further as the SRF model is more flexible than the SRI model with respect to the dependence structure of panel outcomes, it can capture the correlation structure implied by the SRI, yielding essentially the same estimates for data set 1. In the SF model the assumption $\mathsf{Cov}(x_i^*, \mathbf{y}_{ji}) = \lambda_x \boldsymbol{\lambda}_j$ can be too restrictive with the covariance being proportional to $\boldsymbol{\lambda}_j$ by factor λ_x and the estimation of $\boldsymbol{\lambda}_j$ driven by the outcome covariances. For data sets 1 and 2, where the correlations to the latent utility are not proportional to $\boldsymbol{\lambda}_j$ for t=1,. . . , 4, the SF model yields biased estimates. Although the SR models allow for more flexible correlation structures, the estimated marginal correlations in data set 3 under these two models also deviate considerably from the true values. In this case the positive definiteness condition for $\mathsf{Cov}(\boldsymbol{\varepsilon}_{ji}, \eta_i)$ restricts the range of possible correlations: Given the dependence structure of $\mathbf{y}_{ji}$, where the latent factor accounts for at least 50% of the composite error, the true marginal correlations between $\mathbf{y}_{ji}$ and x_i^* could be obtained in an SR model only with very high correlations between the pure error of the outcomes and the error in the latent utility model. For instance, the value for correlation parameter $\boldsymbol{\rho}_0$ yielding the true marginal correlations would be $\boldsymbol{\rho}_0 \approx (0.69, 0.69, 0.57, 0.57)$ which is far beyond the region $\sum_{t=1}^{T} \rho_{j,t}^2 < 1$ required for positive definiteness.

5.2 Results for Average Treatment Effects

Figure 2 shows the true and the estimated average treatment effects for the three data sets that were obtained under the different model specifications. As expected, the estimated effects are almost identical to the true values if the correct model is employed for inference and we again observe that the SRF model yields almost identical estimates to the SRI model for data set 1. In these cases the true values of all the model parameters, including the covariance and correlations parameters, were well recovered. However, in the remaining cases the biased parameter estimates resulting from a model mis-specification discussed above have implications for the estimation of the Average Treatment Effects. Figure 2 shows that these incorrect model based estimates exhibit larger deviations from the true effects. If the regression effects in the potential outcome models are biased and the dependence structure between outcome and latent utility cannot be captured by the employed model, the estimates of the ATE are negatively affected.



(a) SRI data

(b) SRF data

(c) SF data

Figure 2: True and estimated average treatment effects in different models for data 1-3

In these cases one or more of the average treatment effects are biased, as can be seen from Table 3. The table reports the difference between the true and estimated average treatment effects. Bold numbers indicate estimates when the true value is not contained in the 99% posterior density interval and slanted bold numbers indicate estimates where the true value is not contained in the 95% posterior density interval. Our results also indicate that capturing the correlation between latent utility and outcome might be more important than the correlation structure within the outcome vectors in the context of the average treatment effects: in data set 2 the SRI model yields only slightly biased estimates compared to those from the SF model.

5.3 Inefficiency factors and estimation issues

The results discussed above show that the sampler performs well, with posterior estimates recovering the true parameter values well if the correct model is used for inference. We now turn to other aspects of the performances for the three samplers.

Table 3: Average Panel Treatment Effects: difference between posterior mean and true value, sd (in parentheses)

data	t	SRI	SRF	SF
1	1	-0.012 (0.014)	-0.009 (0.014)	**-0.124** (0.011)
	2	-0.001 (0.015)	-0.007 (0.017)	**-0.084** (0.011)
	3	-0.012 (0.014)	-0.008 (0.014)	**-0.051** (0.011)
	4	-0.007 (0.016)	-0.009 (0.018)	***-0.026*** (0.011)
2	1	-0.014 (0.012)	-0.003 (0.011)	**-0.056** (0.008)
	2	-0.020 (0.013)	0.001 (0.013)	**-0.046** (0.010)
	3	-0.010 (0.012)	-0.003 (0.011)	**-0.056** (0.008)
	4	***0.031*** (0.013)	0.015 (0.013)	-0.011 (0.010)
3	1	0.022 (0.017)	0.021 (0.017)	-0.015 (0.009)
	2	**0.058** (0.018)	**-0.049** (0.021)	-0.009 (0.010)
	3	0.028 (0.017)	0.021 (0.017)	-0.016 (0.009)
	4	***0.044*** (0.018)	***0.039*** (0.018)	-0.011 (0.010)

For the SF model inefficiency factors are satisfactory for all parameters. As noted in Appendix B, the marginal variance of the latent utility is $1+\lambda_x^2$ in the SF model, and therefore only parameters in the rescaled probit model can be compared to the SR model. Whereas regression effects in the selection equation as well as the factor loading λ_x can suffer from high autocorrelations, the parameters in the rescaled probit model exhibit small inefficiency factors from 2 to 11.

Inefficiency factors are generally higher in the SR models estimates. While the sampler for the SF model requires only Gibbs steps due to the simpler modeling of the correlation between latent utility and outcomes, the SR models involve MH-steps usually associated with higher autocorrelation in the chain. In particular the inefficiency factors for the correlation parameters are large, leading to inefficient estimation of the marginal correlations (the highest inefficiencies were observed in data set 1 with up to roughly 300 for the SRI and the SRF model). Also, draws of the regression effects $\boldsymbol{\alpha}$ in the selection equation show higher autocorrelations than under the SF model (inefficiency factors up to 40-50 in SRI and the SRF model).

Finally, due to the simpler sampling scheme computation is much faster for the SF model than for the SR models by a factor of roughly 20, with the SRF model being the computationally the most intensive model to fit.

6 Application

6.1 Data and Sample

The data for our analysis comes from two data sets, one is the Austrian Social Security Data Base (ASSD), which is an administrative data set of the universe of Austrian employees and provides detailed information on employment spells and maternity leave spells, as well as demographic information on mothers and information on employers. The second is a data set collected as basis for wage taxes.

For our analysis we focus on women that gave birth within a 4 years period around the change in the parental leave policy in July 2000 and consider those who gave birth between July 1998 and June 2002. This period was chosen to (i) create a sample with a balanced window of mothers before and after the policy change, and (ii) to ensure that we can observe a reasonable number of periods for each mother after the end of her return to the labor market for our panel analysis. For women who have more than one child between July 1998 and June 2002 we will consider the last child birth in the period.

To create a comparable sample of mothers and their earnings after reentry, we restrict our attention to mothers who returned to the labor market after the end of the maternity leave (or more precisely within 30 days after the end of maternity leave). Additionally we restrict the data to include only mothers with earnings above 1100 Euro per year. We further focus on women that were employed in the private sector in the year before child birth to ensure eligibility for the standard maternity leave policy regimes in place at the time. This restriction also enables us to compute a baseline earnings variable to account for the earnings level before the birth of the child (first child if more that one in the considered period).

Finally, to ensure the identification of common year effects and panel effects separately by treatment status we consider mothers who return from 2000 until the year 2008 and for whom we have at least 4 consecutive panel observations. Table 4 shows the distribution of the data in the sample by (contribution) year and panel period for both treatment groups.

year	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$	total*
2000	825/207	0/0	0/0	0/0	0/0	0/0	1,032
2001	5,676/381	825/207	0/0	0/0	0/0	0/0	7,089
2002	5,985/697	5,676/381	825/207	0/0	0/0	0/0	13,771
2003	1,088/4,129	5,985/697	5,676/381	825/207	0/0	0/0	18,988
2004	457/7,179	1,088/4,129	5,985/697	5,676/381	825/206	0/0	26,622
2005	84/4,343	457/7,179	1,088/4,129	5,985/697	5,676/381	820/206	31,026
2006	0/0	84/4,343	457/7,179	1,088/4,129	5,985/694	5,643/379	29,960
2007	0/0	0/0	84/4,343	457/7,179	1,086/4,114	5,943/693	23,899
2008	0/0	0/0	0/0	84/4,343	454/7,109	1,075/4,087	17,152
total	14,115/16,936	14,115/16,936	14,115/16,936	14,115/16,936	13,985/12,504	13,481/5,365	169,539

Table 4: Distribution of mothers in the sample by contribution year and panel period for each treatment group. total* refers to $x = 0$ and $x = 1$ observations.

The sample restrictions result in an unbalanced sample of 31,051 mothers that are observed over 4-6 consecutive panel periods, i.e. have no breaks in their employment histories (working at least 360 days the year following the end of the maternity leave). Since we based our analysis on yearly earnings we define the first earnings observation for the year after their return when they report to work at least 360 days.

In Table 5 we present some summary statistics for the sample. Overall 58% of mothers in the sample went on leave under the new leave policy. Most mothers have either one child (49.7%) or two children (40.8%), and of the remaining almost all have 3 children. Based on the distribution of the number of children we define a dummy variable for having two children and a dummy variable for having more than 2 children.

An interesting point to note is that while overall 58% of the mothers took leave under the new policy, the proportion is 13% of the mothers with short leave and 95% of the mothers with long leave. Again, this confirms the large positive impact of the increase in the maternity benefit period from 18 months to 30 months on the lengths of leave taken by mothers.

6.2 Model Specifications

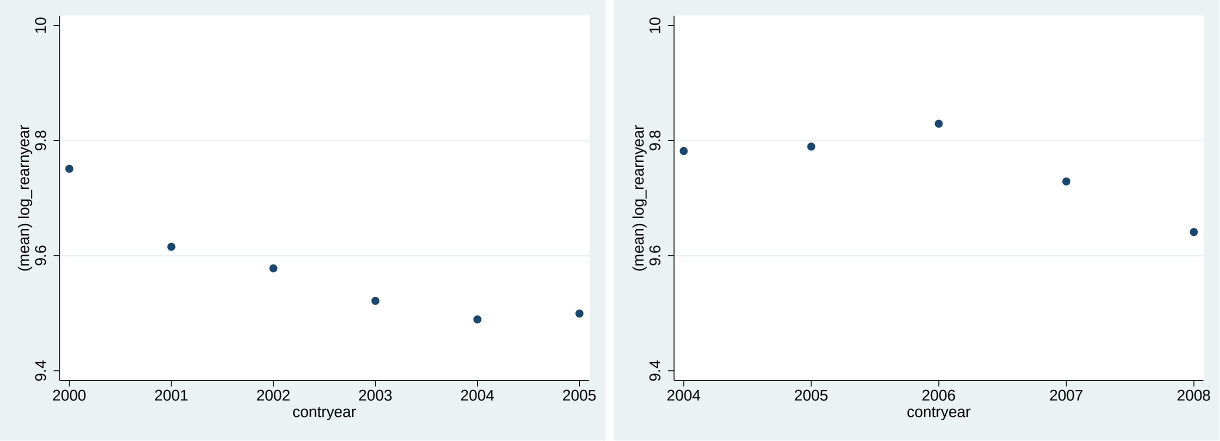
We now specify the covariate vectors for the selection and potential outcome models for the analysis of the earnings effects based on the unbalanced sample of mothers described in the

	Overall Sample		$x = 0$	$x = 1$
Variable	mean	sd	mean	mean
z	0.58		0.13	0.95
age mother	30.47	4.88	30.45	30.49
number of children	1.62	0.71	1.62	1.61
working experience (at birth)	9.39	4.58	9.24	9.51
blue collar	0.31		0.32	0.29
same employer	0.74		0.80	0.69
real earnings* base year	20689.44	9840.47	20776.58	20616.81
real earnings* year 1	15997.88	8719.40	17603.46	14659.74

Table 5: Selected sample summary statistics. * in EUR

previous section under switching regression models and the shared factor model. For the selection model into the long leave treatment we specify the covariate vector $\mathbf{Z}$ to include demographic control variables and controls referring to the labor market experience of mothers before maternity leave: two indicator variables for 2 children or more than two children; an indicator variable for high work experience (above median) before maternity leave; an indicator for blue collar; and a control for earnings before maternity leave for first child or (if more than 10 years since birth of first child year based on earnings before the birth of last child) in terms of indicators for baseline earnings quartiles.

These controls are also included in the potential outcome models, as well as an indicator whether a mother returns to the same employer. In addition we include flexible controls for the panel periods and a quadratic time trend to account for two specific features of the data, strong panel period and year effects. Figures 3 and 4 illustrates these data features. The graphs of average log earnings by year holding the panel periods fixed at 1 or 4 in panels (a) and (b) of Figure 3 respectively point to clearly present and potentially non-linear calender year effects in the data.

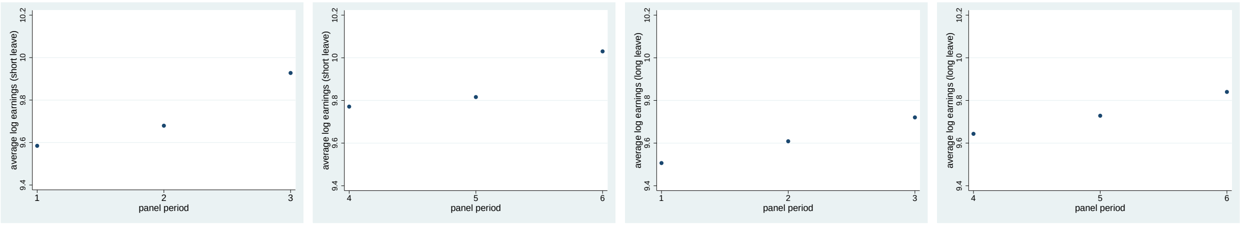


(a) Year Effects (Panel Period 1)

(b) Year Effects (Panel Period 4)

Figure 3: Average log earnings by year with panel period held fixed.

The graphs in Figure 4 show the average log earnings by panel period for short leave



(a) $x = 0$ group (2002) **(b)** $x = 0$ group (2005) **(c)** $x = 1$ group (2005) **(d)** $x = 1$ group (2008)

Figure 4: Average log earnings by panel period with year fixed.

mothers (panels (a) and (b)) and long leave mothers (panels (c) and (d)) holding the calender year fixed. Due to the structure of the data (see Table 4) we graph the average log earnings for three panel periods in a given year, which ensures a reasonable number of observations for each panel period and treatment group. While we need to keep in mind that the figures are based on different years and thus not directly comparable, and reflect raw earnings summaries without controlling for any other factors, they are a strong indicator that mothers' earnings increase substantially in the years (panel periods) following their return to the labor market. The most likely reason behind this pattern is that mothers increase their working hours as their child gets older, partially driven by the better availability of child care options for older children. Since the data do not include information on working hours, this effect is reflected in an increase in yearly earnings. Also, the panel effects are likely to differ across the two treatment groups, for example due to the fact that mothers with a short leave return when their child is on average younger compared to mothers returning after a long leave.

In order to capture the year effects, which are assumed to be common across the treatment groups with mothers facing the same labor market, and dynamic panel effects by treatment group we include a quadratic time trend (common across both potential outcome sequences and excluded from variable selection) and indicators for the panel periods in the potential earnings models, so that the means of potential outcome models are specified as

$$\begin{aligned}\mu(\mathbf{y}_{0i}) &= \mathbf{1}_T\mu + \mathbf{W}_i\boldsymbol{\gamma}_1 + \mathbf{I}_{Pi}\boldsymbol{\gamma}_2 + c\gamma_3 + c^2\gamma_4,\\ \mu(\mathbf{y}_{1i}) &= \mathbf{1}_T(\mu+\kappa) + \mathbf{W}_i(\boldsymbol{\gamma}_1 + \boldsymbol{\theta}_1) + \mathbf{I}_{Pi}(\boldsymbol{\gamma}_2 + \boldsymbol{\theta}_2) + c\gamma_3 + c^2\gamma_4,\end{aligned}$$

where $\mathbf{I}_{Pi}$ refers to the matrix of indicators for panel periods 2 to 6 (panel period 1 is base category) and c is calendar year - 2000.

6.3 Results

In the following we present the results from our prior-posterior analysis of mothers' earnings after a short and long maternity leave under the three modeling approaches discussed in the paper, the two switching regression model specifications with a random intercept (SRI) or latent factor (SRF) and the shared factor model (SF). Bayesian inference was implemented both with and without variable selection. In the discussion below we focus on the model specifications with stochastic variable selection. All results are based on 10,000 runs of the corresponding MCMC algorithm following a burn-in period of 10,000 iterations for which the draws are discarded to allow for convergence of the sampler.

6.3.1 Selection and Earnings Model Estimates with Variable Selection

We first present the results on the parameter estimates for the selection model into the maternity leave treatment for all three models. Table 6 below reports the posterior means and standard deviations as well as the estimated inclusion probabilities for the two versions of the switching regression model (SRI and SRF) and the shared factor model (SF).

Table 6: Results selection equation: posterior means, sd (in parentheses) and posterior inclusion probabilities of regression effects; *estimated inclusion probability, no selection on intercept; *results for SF with variance adjustment*

	SRI Model		SRF Model		SF Model	
	mean (sd.)	prob*	mean (sd.)	prob*	mean (sd.)	prob*
intercept	-1.529 (0.033)	–	-1.540 (0.029)	–	-1.575 (0.032)	–
z	2.792 (0.023)	1.000	2.793 (0.026)	1.000	2.825 (0.022)	1.000
child 2	0.025 (0.034)	0.396	0.021 (0.030)	0.366	0.051 (0.037)	0.713
child ≥ 3	-0.018 (0.038)	0.217	-0.026 (0.045)	0.287	-0.015 (0.036)	0.186
experience	0.072 (0.032)	0.899	0.091 (0.023)	0.996	0.101 (0.025)	0.997
blue collar	-0.066 (0.042)	0.778	-0.043 (0.042)	0.568	-0.027 (0.038)	0.382
int exp./ blue	-0.010 (0.033)	0.111	-0.018 (0.020)	0.198	-0.018 (0.040)	0.206
base-earn Q2	0.001 (0.007)	0.025	0.002 (0.011)	0.055	0.002 (0.011)	0.051
base-earn Q3	-0.000 (0.004)	0.018	-0.001 (0.005)	0.024	-0.001 (0.006)	0.025
base-earn Q4	-0.131 (0.028)	0.999	-0.135 (0.026)	1.000	-0.150 (0.026)	1.000

In all three models we observe a strong positive effect of the policy change and work experience before the maternity leave for selection into long maternity leave, while having base earnings in the highest quartile has a negative effect for selection into long leave. Having one child already has a positive effect on selection into long leave under the SF model, while the estimated effect is much lower and not selected under the switching regression models, with inclusion probability of 0.396 and 0.366. Being a blue collar worker is found to have a negative effect on taking up a long leave under the SRI model. In the other two models the coefficient estimate is also negative but smaller with the estimated inclusion probabilities even below 0.5 in the SF model. Having more than two children or baseline earnings in the 2nd and 3rd quartile appear to have no effect on selection into the long maternity leave treatment, the same is found for the interaction of experience and being a blue collar worker.

Table 7 reports the estimates of the intercept and the covariate effects, that can vary by treatment state, in the earnings model. For each model the first column presents the posterior means and standard deviations on the common effect (μ,γ_1,γ_2) and the second column for the additional effect under the long leave treatment (κ,$\boldsymbol{\theta}_1$,$\boldsymbol{\theta}_2$). For a better reading of the table estimated inclusion probabilities above 0.5 are indicated by "*". First we notice that the three models agree in terms of which covariates affect the yearly earnings (selected into the model): experience, being a blue collar worker, having base earnings above the 1st the quartile, returning to the same employer and the panel period, as well as the sign of their effect. An interesting feature is the steady increase in the effects of the panel periods. While the exact magnitude of the effects varies slightly across the three models, the effect roughly triples from the 2nd to the 6th period. One obvious explanation for this pattern is that mothers increase the hours they work as the (youngest) child gets older and more child care options

Table 7: Results Earnings Model: posterior means, sd (in parentheses); inclusion of regression effects based on posterior inclusion probabilities > 0.5 indicated by "*"

	SRI Model		SRF Model		SF Model	
	treatment 0	+ treatment 1	treatment 0	+ treatment 1	treatment 0	+ treatment 1
intercept	9.351 (0.014)	-0.141 (0.013)*	9.334 (0.012)	-0.125 (0.011)*	9.329 (0.011)	-0.109 (0.010)*
child 2	-0.000 (0.001)	-0.000 (0.002)	-0.000 (0.001)	-0.000 (0.001)	-0.000 (0.001)	-0.000 (0.001)
child >3	0.000 (0.001)	0.000 (0.002)	0.000 (0.001)	0.000 (0.002)	0.000 (0.000)	0.000 (0.002)
exp	-0.096 (0.008)*	0.004 (0.010)	-0.087 (0.008)*	0.005 (0.010)	-0.086 (0.007)*	0.003 (0.009)
blue collar	-0.120 (0.007)*	0.000 (0.002)	-0.103 (0.006)*	0.000 (0.015)	-0.102 (0.007)*	0.000 (0.002)
int. exp/blue	0.002 (0.008)	0.016 (0.018)	0.001 (0.004)	0.007 (0.013)	0.001 (0.003)	0.007 (0.013)
base-earn Q2	0.064 (0.007)*	0.000 (0.002)	0.068 (0.006)*	0.000 (0.002)	0.069 (0.006)*	0.000 (0.002)
base-earn Q3	0.281 (0.011)*	-0.039 (0.016)*	0.292 (0.010)*	-0.049 (0.012)*	0.291 (0.009)*	-0.047 (0.012)*
base-earn Q4	0.609 (0.010)*	-0.106 (0.013)*	0.615 (0.010)*	-0.117 (0.012)*	0.611 (0.009)*	-0.117 (0.012)*
eq. emp.	0.040 (0.005)*	0.000 (0.001)	0.051 (0.005)*	0.001 (0.003)	0.051 (0.005)*	0.000 (0.001)
panel t=2	0.070 (0.006)*	0.099 (0.007)*	0.071 (0.006)*	0.095 (0.007)*	0.073 (0.005)*	0.061 (0.005)*
panel t=3	0.117 (0.007)*	0.125 (0.006)*	0.116 (0.007)*	0.118 (0.006)*	0.118 (0.006)*	0.094 (0.006)*
panel t=4	0.165 (0.010)*	0.146 (0.007)*	0.162 (0.009)*	0.139 (0.007)*	0.163 (0.008)*	0.107 (0.005)*
panel t=5	0.219 (0.012)*	0.151 (0.007)*	0.217 (0.012)*	0.142 (0.007)*	0.215 (0.010)*	0.113 (0.006)*
panel t=6	0.271 (0.014)*	0.181 (0.008)*	0.267 (0.014)*	0.169 (0.008)*	0.262 (0.012)*	0.132 (0.007)*
$(\text{year} - 2000)$	0.039 (0.004)		0.036 (0.004)		0.033 (0.004)	
$(\text{year} - 2000)^2$	-0.004 (0.0002)		-0.004 (0.0002)		-0.004 (0.0002)	

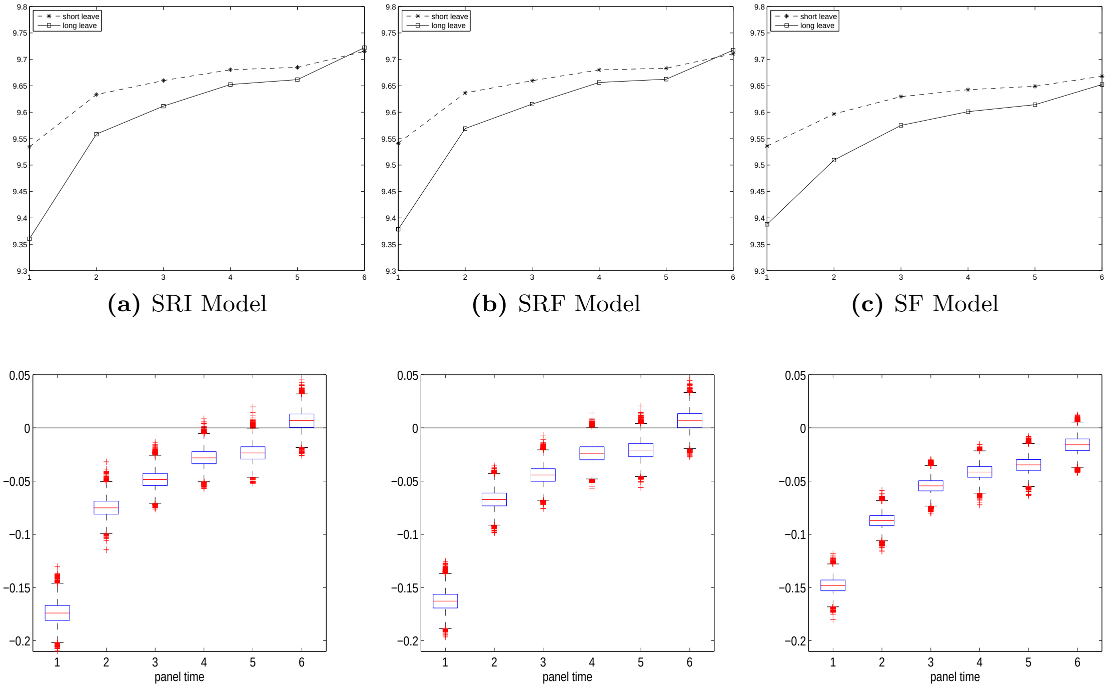
become available. Since our outcome variable is yearly earnings it subsumes any effects of changes in working hours. As expected we observe strong positive effects of having higher base earnings (especially in the highest two quantiles), and returning to the sample employer, and a negative effect for mothers who are blue collar workers. Interestingly experience seems to have a negative effect on earnings here but that is likely a result of not being able to control for hours and mothers with more experience deciding and being able (to afford) to increase their hours more slowly after returning to work.

Second, we observe that all three models indicate an additional effect under the long treatment for the intercept as well as having base earnings in the 3rd and 4th quartile and the panel period. Again we observe the same sign of the additional effects under long leave in the three models, with a negative constant earnings effect, an earnings penalty for mothers with base earnings in the third and fourth quartile as well as higher panel effects in all periods under long leave. These additional effect contribute to the heterogeneous treatment effect. We do notice some variation in the exact magnitude of the panel effects for long leave mothers (as we did in the common panel effects), but we again observe a steady increase in the coefficients over the panel periods. This seems to again reflect at least in part the increase in work hours, as the child gets older. Finally, the three models yield almost identical estimates of the quadratic year effects.

Overall, the above results point to the importance of a flexible modeling of the dynamic panel effects and presence of heterogeneous treatment effect rather than just a constant treatment effect commonly assumed in the literature. The results also show that the specifics of the different model specifications have some effect on the size of the coefficients and estimated inclusion probabilities. The differences across the models will be more visible in the next section when we look at the covariance and correlations structures implied by the three models.

6.3.2 Earnings Dynamics and Treatment Effects

In this section we present results on the potential earnings dynamics and the earnings effects from the three models. The upper panel in Figure 5 graphs the potential log earnings dynamics in terms of the posterior means of the potential log earnings distributions implied by the three models, the lower panel gives box plots of the average treatment effects in terms of log earnings. In all three models we observe that mothers with a long leave start out with considerably lower potential earnings than mothers with a short leave in the first period of their return to the labor market, with the gap decreasing over the next 4-5 panel periods. All three models thus suggest that long leave mothers eventually catch up with those who took a short leave.



(a) SRI Model **(b)** SRF Model **(c)** SF Model

(d) SRI Model **(e)** SRF Model **(f)** SF Model

Figure 5: Dynamics of average potential log earnings for short and long leave mothers (upper panel) and average treatment effects under the two switching regression models and the shared factor model (lower panel).

However, the exact dynamics of the potential earnings vary across the models. The two specifications of the switching regression model imply that long leave mothers close the gap entirely and catch up with short leave mothers 5 years after the return. The shared factor model implies a smoother path of the average earnings over time and more steady reduction in the earnings gap between long and short leave mother. Different from the SR model it does not suggest that long leave mothers fully catch up with short leave mothers over the 6 year period following the return to the labor market. These patterns are also reflected in the estimates for the average earnings effects in terms of log earnings and percentage changes reported in Table 8. Mothers with long leave return with a gap of roughly 0.15-0.17 in log earnings under all three models, which implies roughly a 15% earnings gap in the first year. After 6 years mothers with a long leave have on average 0.8% higher earnings under the two SR models, and

Table 8: Average treatment effects for the three models.

t	ATE log income: mean (sd)			ATE percentage change: mean (sd)		
	SRI Model	SRF Model	SF Model	SRI Model	SRF Model	SF Model
1	-0.174 (0.010)	-0.163 (0.010)	-0.148 (0.008)	-15.9 (0.9)	-14.9 (0.8)	-13.7 (0.7)
2	-0.075 (0.009)	-0.067 (0.009)	-0.087 (0.007)	-7.1 (0.9)	-6.4 (0.9)	-8.2 (0.7)
3	-0.048 (0.009)	-0.044 (0.009)	-0.054 (0.007)	-4.6 (0.8)	-4.2 (0.8)	-5.2 (0.7)
4	-0.028 (0.008)	-0.024 (0.009)	-0.041 (0.007)	-2.7 (0.8)	-2.2 (0.8)	-3.9 (0.7)
5	-0.023 (0.009)	-0.021 (0.009)	-0.035 (0.008)	-2.2 (0.9)	-1.9 (0.9)	-3.3 (0.7)
6	0.007 (0.009)	0.007 (0.010)	-0.016 (0.008)	0.8 (1.0)	0.8 (1.0)	-1.4 (0.8)

a remaining gap of 1.4% under the SF model. However the 95% HPD intervals for the average treatment effects include zero under the switching regression model, and the upper boundary of this interval is close to zero in the factor model.

The differences in the potential earnings dynamic and treatment effect patterns are to a large extend driven by the different panel effects estimated for both treatment groups under the three different models that were reported in Table 7.

Under the shared factor model we can also estimate a further set of treatment effects, the treatment effect on the treated (TT) and the treatment effect on the untreated (TU), as defined in equations (3.17) – (3.18), where the former refers to the average earnings effects of a mother with long leave and the latter to the average earnings effects of mothers with short leave. Figure 6 shows the posterior mean and the 95% HPD-interval (indicated by vertical bars) of the TT (solid blue line) and the TU (dashed blue line) and for comparison also the ATE (black line). While the TT is essentially an upward a shift of the ATE, the TU is a downward shift of the ATE with the spread between the two slightly increasing over time. Both effects start out negative in the first panel period and are decreasing over time, however the TT is positive from the 4th panel period, while the TU remains negative until the last period. These estimates suggest that mothers taking short leave would have suffered a considerable higher earnings penalty under long leave than those mothers who chose long leave, as a result of selection based on observable and unobservable characteristics. For example, we know from the estimation results that mothers with earnings in the highest quartile before child birth are

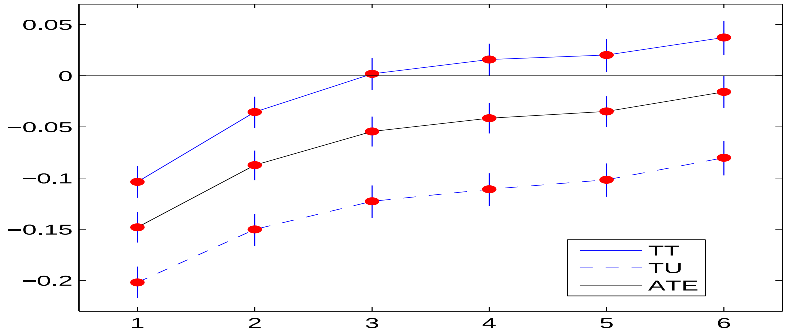


Figure 6: Treatment effects on treated and untreated under the shared factor model.

more likely to decide against a long leave and would suffer a earnings penalty under long leave as indicated by the negative modified effect. The positive selection on unobservables might be at least partially a result of adjustments in the labor market to most mothers returning after a longer leave under the new more generous policy regime.

6.3.3 Variances, Covariances and Correlation Structures

We next present the results for the correlation between the latent utility associated with the maternity leave treatment and the two potential earnings. As discussed previously, the three models differ in their specification of the correlation structure between the latent utility and the potential outcomes and the correlation structure within the potential earnings due to the role of the latent factor. Hence, as in section 5.1.2 we compare the correlations from the shared factor model with those of the switching regression models marginalized over the random intercept and the latent factor (equations (5.1), (5.2) and (3.13)). The estimates are given in Table 9. All three models imply similar patterns with a negative correlation between latent treatment and the potential earnings under short leave for all periods, and a positive correlation for potential earnings under long leave for all periods. In other words, we have negative confounding between utility associated with longer leave and earnings under long leave for short leave mothers or positive confounding between utility associated with shorter leave and earnings under short leave, i.e. mothers under short leave gain from short leave. For mothers in the long leave group the positive confounding indicates a "gain" on long leave from the confounding factors. One interpretation of this finding is that mothers chose the best option given their circumstances, such as by us unobserved options regarding flexible working hours and their intentions regarding working hours.

Table 9: Marginal correlations between latent utility and outcomes, posterior means, sd (in parentheses)

	SRI Model		SRF Model		SF Model	
t	treatment 0	treatment 1	treatment 0	treatment 1	treatment 0	treatment 1
1	-0.119 (0.012)	0.279 (0.023)	-0.117 (0.012)	0.224 (0.023)	-0.176 (0.009)	0.170 (0.008)
2	-0.162 (0.011)	0.084 (0.022)	-0.157 (0.009)	0.011 (0.026)	-0.206 (0.010)	0.218 (0.011)
3	-0.185 (0.008)	0.192 (0.018)	-0.181 (0.008)	0.141 (0.021)	-0.225 (0.011)	0.238 (0.012)
4	-0.196 (0.005)	0.145 (0.021)	-0.194 (0.005)	0.093 (0.023)	-0.236 (0.012)	0.231 (0.011)
5	-0.203 (0.006)	0.181 (0.021)	-0.197 (0.007)	0.127 (0.023)	-0.234 (0.011)	0.219 (0.011)
6	-0.184 (0.008)	0.112 (0.020)	-0.180 (0.008)	0.068 (0.023)	-0.222 (0.011)	0.210 (0.010)

The magnitude of the correlations is similar for the short leave treatment in all three models, with more pronounced correlations in the shared factor model. Under long leave treatment the shared factor model yields positive correlations around 0.2, whereas the other two models yield only low positive correlations for $t = 2$. The more flexible modeling of the correlation structures in the switching regression models might be responsible for the higher variation across the time periods, while the structure of the shared factor model induces a temporal smoothing of the correlations across time. It should also be noted that, in particular in the two switching regression models, the correlations are identified based on mothers that chose a maternity leave length different from the incentives given by the policy regime in place. This is a very small subset of 772 mothers who chose long leave prior to policy change. Under the SF model the correlations are in part based on the estimates of the factor loadings that are more embedded in the modeling structure with more identification coming from the model in addition to the smoothing from the λ_x component.

The estimated variances in the outcome model are essentially identical across the three model specifications. Under the SRF model the posterior means and standard deviations of the idiosyncratic error terms $\boldsymbol{\epsilon}_{ji}$ in the potential earnings models are as follows:

$$\begin{aligned}\boldsymbol{\sigma}_0^2 &= \{0.123\,(0.002), 0.072\,(0.001), 0.044\,(0.001), 0.026\,(0.000), 0.028\,(0.001), 0.046\,(0.001)\},\\ \boldsymbol{\sigma}_1^2 &= \{0.102\,(0.001), 0.044\,(0.001), 0.019\,(0.000), 0.028\,(0.000), 0.045\,(0.001), 0.058\,(0.001)\}.\end{aligned}$$

An interesting feature is the decrease in variance over the first 3-4 panel periods and a slight increase afterwards. The decrease is likely to be driven by a convergence in the hours worked by mothers as their children get older and child care availability increase. Similarly, the increase coincides roughly with the onset of the school age, around period 5 under short leave and at period 4 for long leave mothers whose children are on average one year older at their return to the labor market. As school ends midday in Austria with no lunch provided many mothers reduce their working hours again.

In the SRI model the correlation across the potential panel earnings is captured by random coefficients, with the variances estimated as $D_0 = 0.164\,(0.002)$ under the short leave and $D_1 = 0.136\,(0.002)$ under long leave. In the SRF and SF models the correlation is captured by a more flexible factor structure with the time-varying factor loadings estimated as

$$\begin{aligned}|\boldsymbol{\lambda}_0| &= \{0.340\,(0.004), 0.379\,(0.003), 0.411\,(0.003), 0.426\,(0.003), 0.413\,(0.003), 0.395\,(0.003)\},\\ |\boldsymbol{\lambda}_1| &= \{0.290\,(0.003), 0.353\,(0.003), 0.385\,(0.002), 0.384\,(0.002), 0.366\,(0.003), 0.356\,(0.004)\},\end{aligned}$$

under the SRF model and as

$$\begin{aligned}|\boldsymbol{\lambda}_0| &= \{0.341\,(0.004), 0.381\,(0.003), 0.413\,(0.003), 0.428\,(0.003), 0.415\,(0.003), 0.397\,(0.003)\},\\ |\boldsymbol{\lambda}_1| &= \{0.290\,(0.003), 0.356\,(0.003), 0.387\,(0.002), 0.386\,(0.003), 0.368\,(0.003), 0.358\,(0.004)\},\end{aligned}$$

under the SF model. We notice that the absolute values of the factor loadings are almost identical across the two models. In connection with the essentially identical estimates of the idiosyncratic error variances, this leads to roughly the same covariance matrix of the potential earnings vectors marginalized over the random effects across these two models. Below we therefore only report the covariance matrices, Ω_j, for SRF model. The estimates of the off-diagonal elements exhibit a reasonable amount of variation, between 0.13 and 0.18 for treatment 0, and between 0.10 and 0.15 under treatment 1. Under both treatments we observe that the correlation between $y_{j,it}$ and the outcomes in the following period increases as t increases. Assuming that mothers increase their work hours, unaccounted for in our analysis due to data limitations, we would expect to see such a pattern. As discussed previously the latent factor structure in the earnings models allows for a more flexible modeling of this covariance matrix. In comparison, the SRI model imposes compound symmetry on the covariance matrices with the off-diagonal terms given by D_j resulting in the following variances

$$\begin{aligned}\mathsf{V}(\mathbf{y}_{0i}) &= \{0.288, 0.234, 0.207, 0.193, 0.194, 0.211\},\\ \mathsf{V}(\mathbf{y}_{1i}) &= \{0.241, 0.178, 0.157, 0.167, 0.182, 0.194\},\end{aligned}$$

Table 10: Covariance matrices under the SRF model, posterior mean.

	treatment 0						treatment 1					
t	1	2	3	4	5	6	1	2	3	4	5	6
1	0.239	0.129	0.140	0.145	0.140	0.134	0.186	0.102	0.111	0.111	0.106	0.103
2	0.129	0.216	0.156	0.162	0.157	0.150	0.102	0.168	0.136	0.136	0.129	0.125
3	0.140	0.156	0.213	0.175	0.170	0.162	0.111	0.136	0.167	0.148	0.141	0.137
4	0.145	0.162	0.175	0.208	0.176	0.168	0.111	0.136	0.148	0.176	0.140	0.137
5	0.140	0.157	0.170	0.176	0.199	0.163	0.106	0.129	0.141	0.140	0.179	0.130
6	0.134	0.150	0.162	0.168	0.163	0.202	0.103	0.125	0.137	0.137	0.130	0.185

and covariances $\mathsf{Cov}(y_{0,it}, y_{0,is}) = \mathsf{0.164}$ and $\mathsf{Cov}(y_{1,it}, y_{1,is}) = \mathsf{0.136}$ after marginalization over the random effects.

Under the SF model the estimated factor loadings of the potential earnings models further imply a correlation structure across the potential outcomes. In Table 11, we report the implied covariance structure between the potential earnings, $\mathsf{Cov}(\mathbf{y}_{0i}, \mathbf{y}_{1i}) = \boldsymbol{\lambda}_0\boldsymbol{\lambda}_1'$. A somewhat puzzling feature of these estimates are the negative signs. According to standard human capital theory, where the correlations are assumed to be driven by unobserved ability, we would expect a positive sign. A high ability mother would be expected to be earning more under short and long leave relative to a low ability mother. While it is not clear what drives the negative correlation in our case, there are a number of possible explanations. An obvious possibility is the lack of information on working hours that would distort the correlation if the work hour patterns are different across the two maternity leave treatments for high (low) ability mothers in that high ability mothers would work less hours under the long leave treatment. One possible scenario that could lead to such a pattern is that high ability mothers who have invested a lot into their career before children, continue with that approach and take a short leave and return for many hours or decide to invest a lot into their child's development by taking a long maternity leave and working less hours after they return. The unobserved difference in working hours would then drive the negative correlation. Another possible explanation is based on assortative matching, i.e. the idea that high earning mothers are married to high earning men, while lower income mothers are more likely to be married to lower income men. Thus for lower income mothers it might not be a financially viable option to return for only a portion of full-time hours due to the lower family income, while high income mothers can rely on their partner's income. Finally, families with high income mothers are also more likely to

Table 11: Shared factor model: Implied covariance between potential outcomes

$\mathsf{Cov}(\mathbf{y}_{0,it}, \mathbf{y}_{1,it})$	1	2	3	4	5	6
1	-0.099	-0.121	-0.132	-0.132	-0.125	-0.122
2	-0.111	-0.135	-0.147	-0.147	-0.140	-0.136
3	-0.120	-0.147	-0.160	-0.159	-0.152	-0.148
4	-0.124	-0.152	-0.166	-0.165	-0.157	-0.153
5	-0.120	-0.148	-0.161	-0.160	-0.153	-0.149
6	-0.115	-0.141	-0.154	-0.153	-0.146	-0.142

have moved away from their parents to pursue their career and have therefore less child care options available while their children are young (no grandparents in town).

7 Conclusion

In this paper we have discussed Bayesian treatment effect models for panel outcomes and investigated the effect of long versus short maternity leave on a mother's earnings in a six-year period following her return to the labor market. For our analysis we have employed a large sample of mothers from the Austrian Social Security Register data and exploited a unique recent change in maternity leave policy. We find substantial but decreasing negative earnings effects from long maternity leave on a mother's earnings over the first 5 years after her return, with the estimated earnings penalties ranging from around 15% in the first period to 2% in the 5th period. We find strong evidence for the presence of heterogeneous treatment effects of the covariates, in particular the panel effects which drive the narrowing of the gap.

To isolate the causal effects of the endogenous maternity leave treatment on later earnings we have introduced two modeling approaches within the potential earnings framework that allow for heterogeneous treatment effects but differ in their assumptions regarding the modeling of two key features, the dependence between the treatment and the outcome and the dependence structure of the panel outcomes. The first modeling framework is based on the switching regression approach and does not impose any assumptions about the (unobserved) joint distribution of the two potential outcome sequences, while the second framework is based on the latent factor approach to model the endogeneity of the treatment. The latter implies assumptions of a joint distribution of the potential outcomes, thus allowing for the estimation of additional treatment effects (treatment effect on the treated and treatment effect on the untreated). An advantage of the latent factor approach is the flexible modeling of the dependence across the panel outcomes compared to the compound symmetry structure imposed under the standard switching regression model with random intercepts. To exploit this flexibility we have introduced a switching regression model that employs latent factors to model the panel dependence of the outcomes, while retaining the flexible modeling of the dependence between the treatment and the outcome under the switching regression setup.

For all three models we have implemented stochastic variable selection on the regression effects via spike and slab priors to test which covariates should be included and to test for the presence of a constant and heterogeneous treatment effects. We have described efficient samplers for each of the models that have been tested in a simulation study, showing also that variable selection can be implemented in the context of treatment models. The simulation study also illustrates potential consequences for model inference as a result from possible mis-specification of the dependence between the treatment and the outcome and the dependence structure within panel outcomes. We find that both the dependence between outcome and latent utility and also the dependence structure within panel outcomes have to be captured correctly to obtain unbiased treatment effect estimates. Inference based on the switching regression model with the more flexible latent factor based modeling of the panel dependence (SRF), rather than the commonly used random intercept with its compound symmetry (SRI), appears to be least affected by mis-specification within our simulation study. A potentially promising extension would be a latent factor model, where one factor captures dependence between latent utility and panel outcomes and one or more factors capture within panel dependence.

A Details on posterior sampling

A.1 Sampling for the Switching Regression Model

The MCMC scheme for posterior inference in the switching regression model with random intercept involves the following steps:

(1) For $i = 1, \dots, n$ sample x_i^* from its conditional posterior distribution, which is the normal distribution given in equation (4.8), truncated to the interval I_{x_i}.

(2) Sample indicator variables, regression effects and random intercepts.

(2a) Sample $(\boldsymbol{\nu}, \boldsymbol{\delta})$ and $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ for the joint regression model with multivariate normal error distribution given in equation (3.7). This step is described in detail in Appendix A.3.

(2b) For $i = 1, \dots, n$ and $x_i = j$ sample the random intercept b_{ji} from the full conditional normal posterior $\mathcal{N}\left(h_i, H_i\right)$ with the posterior moments depending on $x_i = j$:

$$H_i = (1/D_j + \mathbf{1}'(\Sigma_j - \boldsymbol{\omega}_j\boldsymbol{\omega}_j')^{-1}\mathbf{1})^{-1},$$
$$h_i = H_i\mathbf{1}'(\Sigma_j - \boldsymbol{\omega}_j\boldsymbol{\omega}_j')^{-1}\tilde{\mathbf{y}}_i,$$

where $\tilde{\mathbf{y}}_i$ denotes the working observations $\tilde{\mathbf{y}}_i = \mathbf{y}_{ji} - \mathbf{W}_{ji}\boldsymbol{\beta} - \boldsymbol{\omega}_j(x_i^* - \mathbf{Z}_i\boldsymbol{\alpha})$.

(3) For $j = 0, 1$ sample D_j from the conditional posterior $\mathcal{G}^{-1}\left(d_{j0} + n_j/2, D_{j0} + \sum_{i:x_i=j} b_{ji}^2/2\right)$ where n_j is the number of subjects with $x_i = j$.

(4) For $j = 0, 1$ and $t = 1, \dots, T$ sample $\log\sigma_{j,t}^2$ from $p(\log\sigma_{j,t}^2|\Theta^{SRI}\backslash\sigma_{j,t}^2, \mathbf{b}, \mathbf{x}^*, \mathbf{y}, \mathbf{x})$. Updates are performed in a random order of $\{1, \dots T\}$.

(5) For $j = 0, 1$ and $t = 1, \dots, T$ sample $\rho_{j,t}$ from $p(\rho_{j,t}|\Theta^{SRI}\backslash\rho_{j,t}, \mathbf{b}, \mathbf{x}^*, \mathbf{y}, \mathbf{x})$. Updates are performed in a random order of $\{1, \dots T\}$.

(6) Sample π_α and π_β from their respective posteriors $\mathcal{B}\left(1 + k_\alpha, 1 + d_\alpha - k_\alpha\right)$ and $\mathcal{B}\left(1 + k_\beta, 1 + d_\beta - k_\beta\right)$ where $k_\alpha = \sum \nu_l$ is the number of selected regressors for the latent utility and $k_\beta = \sum \delta_l$ accordingly the number of selected regressors for the potential outcomes.

Note that the full conditionals in sampling steps (4) and (5) only involve subjects i with $x_i = j$, see equations (4.9) and (4.10). In both steps we use the Metropolis-Hastings algorithm, where our proposal distribution is a t-distribution with parameters obtained from few maximization steps (currently we use 10 iterations of the SQP algorithm implemented in Matlab). This proposal is truncated to the stationarity region when sampling the correlation parameters, more precisely we propose a value $\rho_{j,t}^*$ from the t-distribution truncated to $\pm\sqrt{0.999 - \sum_{t \neq t^*} \rho_{j,t}^2}$.

For the SRF model with factor structure in the joint variance-covariance matrix $\Omega_j, j = 0, 1$, step (2b) is replaced by sampling the latent factors and step (3) by sampling the factor loadings from their respective full conditionals. These are standard steps in the linear normal model

$$\tilde{\mathbf{y}}_i = \tilde{b}_{ji}\boldsymbol{\lambda}_j + \boldsymbol{\varepsilon}_{ji}, \qquad \boldsymbol{\varepsilon}_{ji} \sim \mathcal{N}_T\left(\mathbf{0}, \Sigma_j - \boldsymbol{\omega}_j\boldsymbol{\omega}_j'\right), \qquad \text{with} x_i = j$$

To take into account non-identifiability of the signs of factor loadings and factors this step is concluded by a random sign-switch. Finally, to sample the parameters $\boldsymbol{\sigma}_j$ and $\boldsymbol{\rho}_j$ we condition on the latent factors $\mathbf{b}$ as well as the factor loadings $\boldsymbol{\lambda}_j$, which is the only modification required in sampling steps (4) and (5).

In detail, the sampling steps which require modification for a factor structure in the outcome covariance matrix are as follows:

(2b*) For $i = 1, \dots, n$ and $x_i = j$ sample the latent factor $\tilde{b}_{ji}$ from the full conditional $\mathcal{N}\left(\tilde{h}_i, \tilde{H}_i\right)$ with moments depending on $x_i = j$:

$$\tilde{H}_i = (1 + \boldsymbol{\lambda}_j'(\Sigma_j - \boldsymbol{\omega}_j\boldsymbol{\omega}_j')^{-1}\boldsymbol{\lambda}_j)^{-1},$$
$$\tilde{h}_i = \tilde{H}_i\boldsymbol{\lambda}_j'(\Sigma_j - \boldsymbol{\omega}_j\boldsymbol{\omega}_j')^{-1}\tilde{\mathbf{y}}_i.$$

(3*) For $j = 0, 1$ sample $\boldsymbol{\lambda}_j^0$ from $\mathcal{N}(\mathbf{l}_j, \mathbf{L}_j)$, where

$$\mathbf{L}_j = (\sum_{i:x_i=j} \tilde{b}_{ji}^2(\Sigma_j - \boldsymbol{\omega}_j\boldsymbol{\omega}_j')^{-1} + \mathbf{L}_{j0}^{-1})^{-1},$$
$$\mathbf{l}_j = \mathbf{L}_j(\sum_{i:x_i=j} \tilde{b}_{ji}(\Sigma_j - \boldsymbol{\omega}_j\boldsymbol{\omega}_j')^{-1}\tilde{\mathbf{y}}_i + \mathbf{L}_{j0}^{-1}\mathbf{l}_{j0}).$$

To perform the random sign-switch of the latent factors $\mathbf{b}$ and the factor loadings $(\boldsymbol{\lambda}_0, \boldsymbol{\lambda}_1)$ sample random variables ξ_j for $j = 0, 1$ with $P(\xi_j = -1) = P(\xi_j = 1) = 0.5$. Set $\boldsymbol{\lambda}_j^{(new)} = \boldsymbol{\lambda}_j\xi_j$ and set $\tilde{b}_{ji}^{(new)} = \tilde{b}_{ji}\xi_j$ for all i, where $x_i = j$ and use $\mathbf{b}^{(new)}, \boldsymbol{\lambda}_0^{(new)}$ and $\boldsymbol{\lambda}_1^{(new)}$ as updated values of the chain. Note that this sign-switch does not change the product $\boldsymbol{\lambda}_j\tilde{b}_{ji}$ for $x_i = j$.

(4*) For $j = 0, 1$ and $t = 1, \dots, T$ sample $\log\sigma_{j,t}^2$ from $p(\log\sigma_{j,t}^2|\Theta^{SRF}\backslash\sigma_{j,t}^2, \mathbf{b}, \mathbf{x}^*, \mathbf{y}, \mathbf{x})$. Updates are performed in a random order of $\{1, \dots T\}$.

(5*) For $j = 0, 1$ and $t = 1, \dots, T$ sample $\rho_{j,t}$ from $p(\rho_{j,t}|\Theta^{SRF}\backslash\rho_{j,t}, \mathbf{b}, \mathbf{x}^*, \mathbf{y}, \mathbf{x})$. Updates are performed in a random order of $\{1, \dots T\}$.

A.2 Posterior Sampling in the Shared Factor Model

In the shared factor model specified in equations (3.9) to (3.11), the error terms $\eta_i, \boldsymbol{\varepsilon}_{0i}, \boldsymbol{\varepsilon}_{1i}$ are independent. Hence the augmented likelihood including the unobserved latent utilities is given as

$$p(\mathbf{x}, \mathbf{x}^*, \mathbf{y}|\Theta^{SF}, \mathbf{f}) = \prod_{i=1}^{n} p(x_i, x_i^*|\boldsymbol{\alpha}, \lambda_x, f_i) \prod_{i:x_i=0} p(\mathbf{y}_{0i}|\boldsymbol{\beta}, \Sigma_0, \boldsymbol{\lambda}_0, f_i) \prod_{i:x_i=1} p(\mathbf{y}_{1i}|\boldsymbol{\beta}, \Sigma_1, \boldsymbol{\lambda}_1, f_i)$$

Conditional on the latent factors, the models for the latent utilities and the potential outcomes are regression models with the additional regressor f_i. This suggests to sample $(\boldsymbol{\alpha}, \lambda_x)$ as well as $(\boldsymbol{\beta}, \boldsymbol{\lambda})$ in one block. To simplify notation we denote by $\boldsymbol{\delta}$ the joint vector of indicators for regression effects $\boldsymbol{\beta}$ and the factor loadings $\boldsymbol{\delta} = (\boldsymbol{\delta}^\beta, \boldsymbol{\delta}^\lambda)$.

The complete sampling scheme involves the following steps:

(1) For $i = 1, \dots, n$ sample the latent factor f_i from the full conditional posterior

$$p(f_i|\Theta^{SF}, x_i^*, \mathbf{y}_{x_i,i}) \propto p(x_i^*, \mathbf{y}_{x_i,i}|\Theta^{SF}, f_i)p(f_i)$$

which is a normal distribution, $\mathcal{N}(f_{n,i}, F_{n,i})$, with the posterior moments depending on $x_i = j$, see equation (4.12).

(2) For $i = 1, \dots, n$ sample x_i^* from $\mathcal{N}(\mathbf{Z}_i\boldsymbol{\alpha} + \lambda_x f_i, 1)$ truncated to the interval I_{x_i}.

(3) Perform variable selection (i.e. sampling of $\boldsymbol{\nu}$) and sample the regression coefficients $(\boldsymbol{\alpha}, \lambda_x)$ in the latent utility model

$$x_i^* = \mathbf{Z}_i\boldsymbol{\alpha} + \lambda_x f_i + \nu_i, \quad \nu_i \sim \mathcal{N}(0,1).$$

Note that only elements of $\boldsymbol{\alpha}$ are subject to selection whereas f_i is not.

(4) Perform variable selection (i.e. sampling of $\boldsymbol{\delta}$) and sampling of regression coefficients $(\boldsymbol{\beta}, \boldsymbol{\lambda}_0, \boldsymbol{\lambda}_1)$ in the model for the observed outcomes $\mathbf{y}_{x_i,i}$, $i = 1, \dots, n$ which is given as

$$\mathbf{y}_{x_i,i} = \mathbf{W}_{x_i,i}\boldsymbol{\beta} + f_i\boldsymbol{\lambda}_{x_i} + \boldsymbol{\epsilon}_{x_i,i}, \quad \boldsymbol{\epsilon}_{x_i,i} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_{x_i}).$$

(5) To take into account non-identifiability of the signs of factors and factor loadings we perform a random sign-switch of $\mathbf{f}$ and $(\lambda_x, \boldsymbol{\lambda}_0, \boldsymbol{\lambda}_1)$, i.e. we sample a random variable ξ with $P(\xi = 1) = P(\xi = -1) = 0.5$, set $\mathbf{f}^{(new)} = \xi\mathbf{f}$, $\lambda_x^{(new)} = \xi\lambda_x$ and $\boldsymbol{\lambda}_j^{(new)} = \boldsymbol{\lambda}_j\xi$ for $j = 0, 1$, and use $\mathbf{f}^{(new)}$, $\lambda_x^{(new)}$ and $\boldsymbol{\lambda}_j^{(new)}$ as updated values of the chain.

(6) For $j = 0, 1$ and $t = 1, \dots, T$ sample $\sigma_{j,t}^2$ from $\mathcal{G}^{-1}(s_{n,jt}, S_{n,jt})$ where

$$s_{n,jt} = s_{0,jt} + n_j/2 \qquad S_{n,jt} = S_{0,jt} + Se_{jt}/2$$

and

$$Se_{jt} = \sum_{i:x_i=j} (y_{j,it} - \mathbf{W}_{j,it}\boldsymbol{\beta} - f_i\lambda_{j,t}^2)^2.$$

Here n_j is the number of subjects with outcome j and $\mathbf{W}_{j,it}$ denotes the values of the covariates at panel time t, i.e. row t of the covariate matrix $\mathbf{W}_{ji}$.

(7) Sample π_α, π_β and π_λ from their respective posteriors $\mathcal{B}(1 + k_\bullet, 1 + d_\bullet - k_\bullet)$ and where $d_\bullet$ is the number of the respective effects subject to selection and $k_\bullet$ is the number of selected effects, i.e. where the corresponding indicators take the value 1.

Sampling steps (3) and (4) are the standard sampling steps used for linear regression models with variable selection and are detailed in Appendix A.3. The modification required in this scheme for sampling for different prior inclusion probabilities π_β and π_λ of the elements of $\boldsymbol{\beta}$ and $\boldsymbol{\lambda}$ is straightforward.

A.3 Variable selection with spike and slab priors in regression models

Consider a linear regression model

$$\mathbf{y}_i = \mathbf{W}_i\boldsymbol{\beta} + \boldsymbol{\varepsilon}_i, \quad \boldsymbol{\varepsilon}_i \sim \mathcal{N}(\mathbf{0}, \mathbf{V}_i),$$

with independent observations $\mathbf{y}_i, i = 1, \dots, n$ and regressor matrix $\mathbf{W}_i$ of dimension $T \times d$. By introducing a $d \times 1$ indicator vector $\boldsymbol{\delta}$ we specify a spike and slab prior distribution for the elements of $\boldsymbol{\beta}$ as

$$p(\boldsymbol{\beta}|\boldsymbol{\delta}) = \prod_{j:\delta_j=1} p_{\text{slab}}(\beta_j) \prod_{j:\delta_j=0} p_{\text{spike}}(\beta_j), \tag{A.1}$$

For elements of $\boldsymbol{\beta}$ which are not subject to selection the corresponding indicator δ_j is set to 1, otherwise $p(\delta_j = 1) = \pi$ with hyper-prior $\pi \sim \mathcal{B}(a_0, b_0)$. Variable selection is based on the posterior probabilities for $p(\delta_j = 1|\mathbf{y})$ which can be sampled using MCMC methods. Here we use a Dirac spike $p_{\text{spike}}(\beta_j) = \Delta_0(\beta_j)$ and specify the slab component by $p_{\text{slab}}(\beta_j) = p(\beta_j|\mathcal{N}(0, B_0))$. Sampling $\boldsymbol{\delta}$ conditional on $\boldsymbol{\beta}$ would result in a reducible Markov chain, and therefore it is essential to sample the indicator vector $\boldsymbol{\delta}$ marginalizing over $\boldsymbol{\beta}$, when a Dirac spike is specified.

For sampling from the posterior distribution the sampling scheme therefore consists of the following steps.

1. Update $\boldsymbol{\delta}$ componentwise in a random permutation ϱ of $(1, \ldots, d)$: For $j = 1, \ldots, d$ set $l = \varrho_j$ and sample δ_l from the posterior

$$p(\delta_l|\boldsymbol{\delta}_{\backslash l}, \mathbf{y}, \pi) \propto p(\mathbf{y}|\boldsymbol{\delta}, \pi)p(\boldsymbol{\delta}|\pi)p(\pi).$$

For a linear regression model the marginalized likelihood $p(\mathbf{y}|\boldsymbol{\delta})$ is available analytically as

$$p(\mathbf{y}|\boldsymbol{\delta}) \propto \frac{|\mathbf{B}_n^{\boldsymbol{\delta}}|^{1/2}}{|\mathbf{B}_0^{\boldsymbol{\delta}}|^{1/2}} \prod_{i=1}^{n} |\mathbf{V}_i|^{-1/2} \cdot \exp\left(-\frac{1}{2}\sum_{i=1}^{n}(\mathbf{y}_i'\mathbf{V}_i^{-1}\mathbf{y}_i) - (\mathbf{b}_n^{\boldsymbol{\delta}})'(\mathbf{B}_n^{\boldsymbol{\delta}})^{-1}\mathbf{b}_n^{\boldsymbol{\delta}}\right),$$

where

$$(\mathbf{B}_n^{\boldsymbol{\delta}})^{-1} = (\mathbf{B}_0^{\boldsymbol{\delta}})^{-1} + \sum_{i=1}^{n}(\mathbf{W}_i^{\boldsymbol{\delta}})'\mathbf{V}_i^{-1}\mathbf{W}_i^{\boldsymbol{\delta}}, \tag{A.2}$$

$$\mathbf{b}_n^{\boldsymbol{\delta}} = \mathbf{B}_n^{\boldsymbol{\delta}} \sum_{i=1}^{n}(\mathbf{W}_i^{\boldsymbol{\delta}})'\mathbf{V}_i^{-1}\mathbf{y}_i, \tag{A.3}$$

and $\mathbf{W}_i^{\boldsymbol{\delta}}$ consists of those columns j of $\mathbf{W}_i$ where the corresponding indicator $\delta_j = 1$ and $\mathbf{B}_0^{\boldsymbol{\delta}} = B_0\mathbf{I}_k$, where $k = \sum_{j=1}^{d} \delta_j$.

2. Sample $\boldsymbol{\beta}^{\boldsymbol{\delta}}$, i.e. the elements of $\boldsymbol{\beta}$ with non-zero indicators from its full conditional posterior $\mathcal{N}_k\left(\mathbf{b}_n^{\boldsymbol{\delta}}, \mathbf{B}_n^{\boldsymbol{\delta}}\right)$ and set the remaining elements of $\boldsymbol{\beta}$ to zero.

3. Denoting by k the number of selected regression effects, i.e. $k = \sum_{j=1}^{d} \delta_j$, the full conditional of π is the Beta-distribution $\mathcal{B}(a_0 + k, b_0 + d - k)$.

B Further results from the simulation study

In Table 12 we report the posterior means and the sampling standard deviations for the regression coefficients in the selection equation. Effects for which the inclusion probability is above 0.5, implying that the corresponding effect is selected into the model are marked with '*'. As the variance of the latent utility σ_x is restricted to 1 in the SR models while the SF model specification implies that $\sigma_x = \sqrt{1 + \lambda_x^2}$, we report the estimation results for $\boldsymbol{\alpha}/\sigma_x$. In data set 3, generated from the SF model, the (true) rescaled regression coefficients are $\boldsymbol{\alpha}/\sqrt{1 + \lambda_x^2} = (-0.74, 0.66, 0, 1.23)$.

For data set 3 the posterior mean of factor loading λ_x was 0.681 (0.021) in the SF model.

Table 12: Results selection equation: posterior means, sd (in parentheses) of regression effects

data		SRI Model	SRF Model	SF Model
1	intercept	-0.91 (0.010)	-0.90 (0.010)	-0.91 (0.010)
	v_1	0.80 (0.008)*	0.80 (0.008)*	0.80 (0.008)*
	v_2	0.00 (0.003)	0.00 (0.002)	0.00 (0.003)
	z	1.51 (0.014)*	1.50 (0.013)*	1.50 (0.014)*
2	intercept	-0.91 (0.010)	-0.90 (0.009)	-0.90(0.010)
	v_1	0.81 (0.008)*	0.80 (0.007)*	0.80 (0.008)*
	v_2	-0.00 (0.002)	0.00 (0.002)	0.00 (0.001)
	z	1.52 (0.014)*	1.50 (0.012)*	1.50 (0.013)*
3	intercept	-0.72 (0.010)	-0.73 (0.008)	-0.72 (0.010)
	v_1	0.66 (0.007)*	0.66 (0.007)*	0.66 (0.007)*
	v_2	0.00 (0.003)	0.00 (0.002)	0.00 (0.004)
	z	1.22 (0.013)*	1.24 (0.012)*	1.22 (0.013)*

Table 13: Difference between true and estimated matrices Ω_0 and Ω_1, sd (in parenthesis)

data		SRI Model				SRF Model				SF Model			
1	Ω_0	-0.002	-0.004	-0.004	-0.004	-0.001	-0.001	-0.004	-0.004	-0.001	-0.006	-0.011	-0.013
		(0.005)	(0.004)	(0.004)	(0.004)	(0.006)	(0.005)	(0.005)	(0.004)	(0.006)	(0.005)	(0.005)	(0.005)
			-0.004	-0.004	-0.004		-0.001	-0.003	-0.002		-0.006	-0.011	-0.013
			(0.005)	(0.004)	(0.004)		(0.006)	(0.005)	(0.005)		(0.006)	(0.005)	(0.005)
				-0.002	-0.004			-0.004	-0.005			-0.015	-0.018
				(0.005)	(0.004)			(0.006)	(0.004)			(0.006)	(0.005)
					-0.003				-0.004				-0.020
					(0.005)				(0.006)				(0.006)
	Ω_1	-0.000	-0.005	-0.005	-0.005	0.001	-0.006	-0.012	0.003	0.064	0.026	0.014	0.019
		(0.016)	(0.010)	(0.010)	(0.010)	(0.019)	(0.014)	(0.014)	(0.014)	(0.019)	(0.012)	(0.013)	(0.013)
			-0.001	-0.005	-0.005		-0.005	-0.016	-0.002		0.027	0.004	0.009
			(0.016)	(0.010)	(0.010)		(0.019)	(0.015)	(0.014)		(0.017)	(0.013)	(0.013)
				-0.011	-0.005			-0.022	-0.007			-0.015	-0.034
				(0.015)	(0.010)			(0.018)	(0.014)			(0.018)	(0.013)
					-0.002				0.005				-0.058
					(0.015)				(0.018)				(0.017)
2	Ω_0	**0.027**	**0.037**	**0.017**	**-0.002**	-0.003	-0.004	-0.002	0.001	-0.008	-0.008	-0.007	-0.003
		(0.003)	(0.002)	(0.002)	(0.002)	(0.004)	(0.003)	(0.003)	(0.002)	(0.006)	(0.004)	(0.003)	(0.003)
			0.014	0.002	**-0.015**		0.002	-0.002	0.001		-0.002	-0.006	-0.003
			(0.003)	(0.002)	(0.002)		(0.003)	(0.002)	(0.002)		(0.004)	(0.005)	(0.005)
				-0.011	**-0.028**			-0.005	0.001			-0.008	-0.002
				(0.003)	(0.002)			(0.003)	(0.002)			(0.003)	(0.002)
					-0.024				-0.001				-0.005
					(0.003)				(0.002)				(0.003)
	Ω_1	**0.082**	**0.125**	0.055	-0.015	0.004	-0.011	0.007	-0.013	0.029	0.002	0.019	-0.004
		(0.013)	(0.006)	(0.006)	(0.006)	(0.015)	(0.011)	(0.009)	(0.009)	(0.014)	(0.010)	(0.009)	(0.008)
			0.026	0.006	**-0.055**		-0.016	-0.002	-0.018		0.004	0.014	-0.006
			(0.012)	(0.006)	(0.006)		(0.014)	(0.009)	(0.009)		(0.013)	(0.008)	(0.008)
				-0.025	**-0.095**			-0.005	-0.005			0.013	0.006
				(0.012)	(0.006)			(0.012)	(0.007)			(0.012)	(0.006)
					-0.05				0.002				-0.001
					(0.012)				(0.011)				(0.011)
3	Ω_0	**0.058**	**0.077**	**0.017**	**0.017**	**0.018**	**0.017**	**0.016**	**0.016**	-0.000	-0.003	0.000	-0.000
		(0.004)	(0.003)	(0.003)	(0.003)	(0.006)	(0.004)	(0.004)	(0.004)	(0.006)	(0.005)	(0.004)	(0.004)
			0.060	**0.017**	**0.017**		**0.019**	**0.015**	**0.015**		-0.002	-0.002	-0.002
			(0.004)	(0.003)	(0.003)		(0.005)	(0.004)	(0.004)		(0.006)	(0.004)	(0.004)
				-0.023	**-0.033**			**0.014**	**0.015**			0.001	0.000
				(0.004)	(0.003)			(0.004)	(0.003)			(0.005)	(0.004)
					-0.025				**0.012**				-0.003
					(0.003)				(0.005)				(0.005)
	Ω_1	**0.024**	**0.061**	0.001	0.001	0.003	0.005	-0.001	0.013	0.003	0.007	0.001	0.015
		(0.013)	(0.008)	(0.008)	(0.008)	(0.014)	(0.011)	(0.010)	(0.010)	(0.013)	(0.009)	(0.009)	(0.008)
			0.011	0.001	0.001		-0.014	-0.006	0.007		-0.012	-0.006	0.008
			(0.013)	(0.008)	(0.008)		(0.015)	(0.010)	(0.010)		(0.013)	(0.008)	(0.009)
				-0.032	**-0.049**			-0.011	0.003			-0.010	0.003
				(0.013)	(0.008)			(0.013)	(0.009)			(0.012)	(0.008)
					-0.025				0.005				0.008
					(0.013)				(0.013)				(0.012)

Table 13 reports the bias (true values – estimated values) for the elements of covariance matrices of the outcome vectors with values not included in the 99%-HPD interval given in bold. Differences are small if the correct (data generating) model is applied to analyze the data. Results for data sets 2 and 3 indicate that the compound symmetry structure implied by a random intercept is too restrictive to capture the dependence structure in models with a latent factor, as deviations of the estimates from the true values can be relatively large.

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